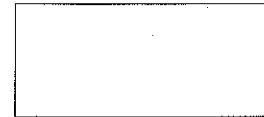


Aspects of Economic Scenario Generation

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Disclaimer

- *This presentation shows the results of a theoretical investigation that have not been published or subjected to formal peer-review. Some of the results are surprising.*
- *Statistically, surprising research results have a higher probability of being incorrect than unsurprising results.*
- *Tentative conclusions drawn from the results represent only my personal view.*

Background

Economic Scenarios and Capital

- Regulatory shift towards economic (i.e., real-world) scenarios with a CTE measure
 - VACARVM, RBC C3-Phase 2
 - C3 Phase 3
 - CALM valuation and Seg Fund Capital
- Elaboration of the older Value-at-Risk risk measure
- Market consistent (risk-neutral) scenarios can be used for valuation; however, they are inappropriate for capital or risk-return related purposes.

Stochastic Valuation Measures

Practical aspects

- Replaces simpler factor-based approaches (capital), or rules-based approaches (reserves).
- Theoretically, should be more appropriate
- Practical issues
 - More complicated
 - Harder to predict
 - *Inherently subject to uncertainty in results*

Economic Scenario Generation Sources of Uncertainty

- | | |
|---|---|
| 1. Choose the functional form of the model (e.g. RSLN) | <ul style="list-style-type: none">• Steps 1-3 each add uncertainty.• There is some regulatory specification to control uncertainty from the first two steps.• Almost all the practical focus has been on the number of scenarios (#3)<ul style="list-style-type: none">– Biggest source of uncertainty or the one we can quantify?• Risk-neutral calibration is much better defined. |
| 2. Calibrate the model's parameters (generally from history) | |
| 3. Create a number (N) of scenarios and run through a valuation model | |
| 4. Calculate the results at a CTE level (generally specified) | |

Research Project Goals and Methodology

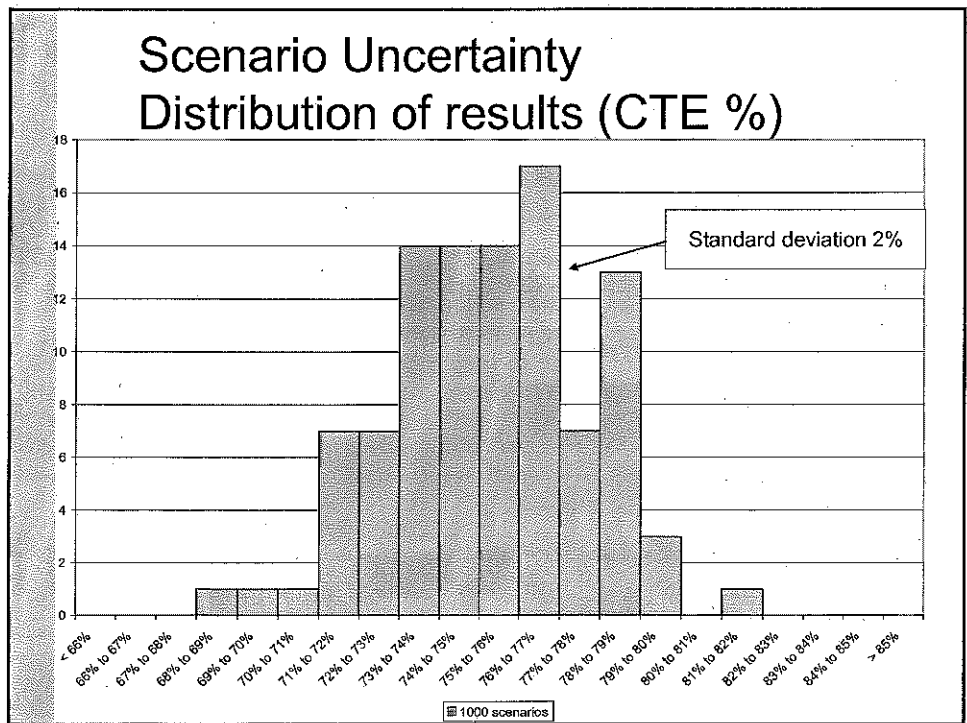
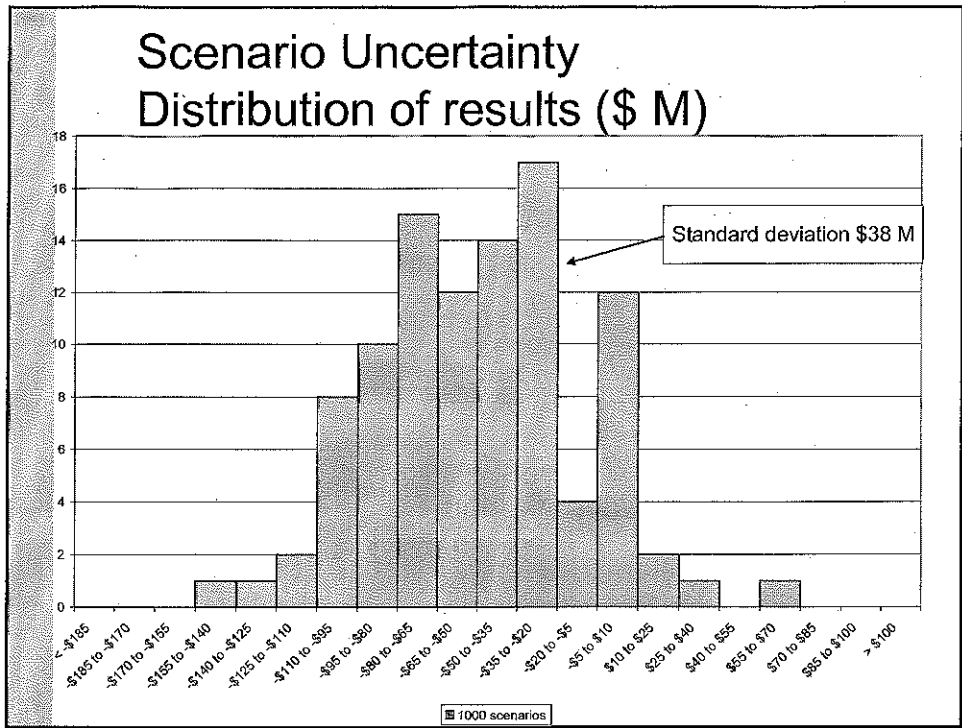
- Goals:
 - Quantify the uncertainty arising from various sources.
 - Develop conclusions related to uncertainty reduction.
- Methodology:
 - Experimental
 - Build a simple liability valuation model and perform huge numbers of runs.

Experimental Model Sample (Quasi-realistic) Liability

- Results intended to be indicative
- Simple VA liability.
 - GMDB with high probability of death.
 - Account Value = \$10.0B
 - Cash Surrender Value = \$9.6 B
- Calculated results
 - PV(fees – expenses – GMDB claims)
 - Compare to the cash surrender value
- No hedging is modeled

Experiment #1 Setting the stage

- Quantification of uncertainty from running a finite number of scenarios.
 - Assume the market is really driven by a true scenario generator. Further assume that we know both the functional form (RSLN) and calibration parameters.
- Experiment #1
 - Run 1,000 scenarios and calculate CTE 75.
 - Repeat 100 times each time using a different seed for the random number generator.



Scenario Uncertainty Comments on quantification

- Experimental quantification:

- standard deviation \$38 M

- Manistre and Hancock formula

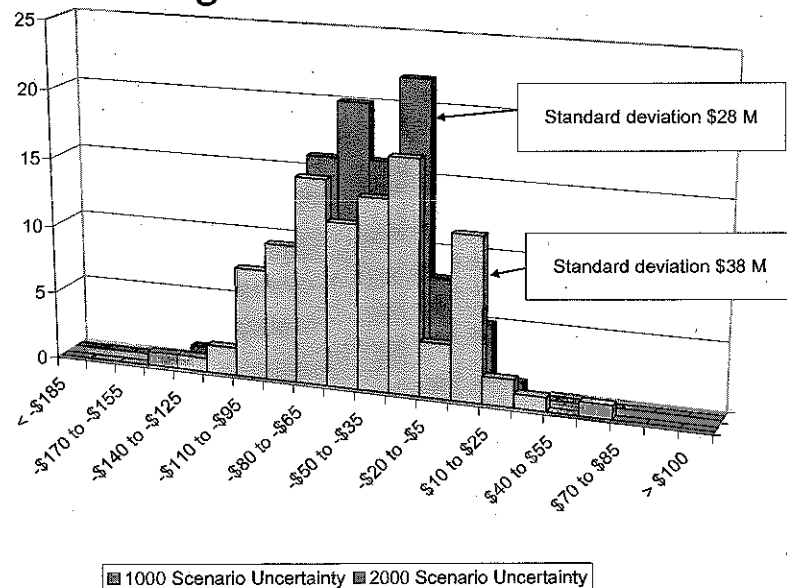
$$Var(C\hat{T}E) \approx \frac{Var(x_{(1)}, \dots, x_{(k)}) + \alpha(C\hat{T}E - x_{(k)})^2}{k}$$

Where $\alpha = 75\%$, $k = 250$ scenarios, and $x_{(i)}$ are the ordered values of the model output.

- Formula → standard deviation \$37 M

- Practical alternative – run 5 sets of 1,000 scenarios (each with its own seed) to validate the formula above (and possibly use this to set the seed).

Scenario Uncertainty Increasing number of scenarios



Another uncertainty reduction method – significance measure

- Method suggested by Chueh
- Choose a significant measure that is calculated based on the scenario (e.g., 10 year S&P return)
 - Generate N scenarios
 - Order them based on significance measure
 - Stratified sample of m of these N scenarios to be used in CFT model
- Value of this method (new formula)

$$\frac{1}{n_{\text{effective}}} \approx \frac{R^2}{N} + \frac{(1-R^2)}{m}$$

- R^2 gives the explanatory power of the significance measure (for the CFT results)
- $n_{\text{effective}}$ is the number of pure random scenarios we would need to run to get equivalent accuracy

Scenario Uncertainty (Tentative) Conclusions

- Scenario-based uncertainty can be quantified
- Scenario-based uncertainty can be reduced
- OPEN QUESTION: Is this uncertainty too large?

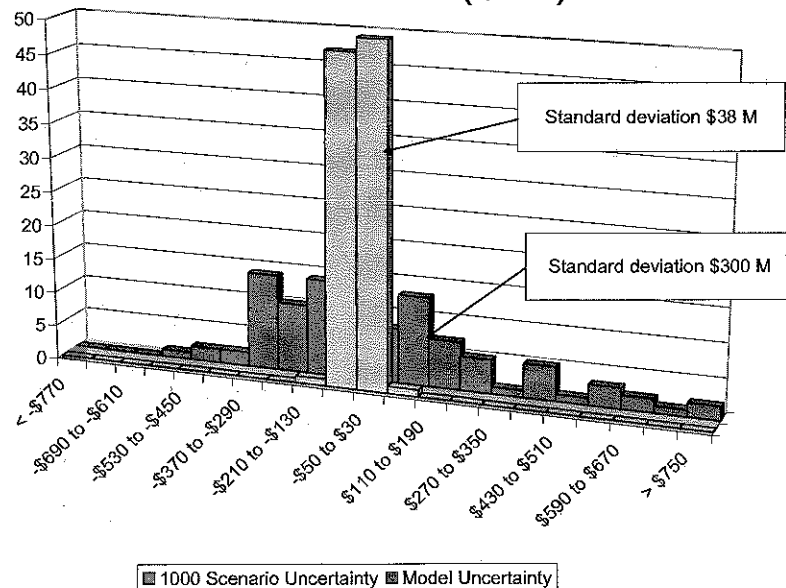
Experiment #2

Quantifying Model Uncertainty

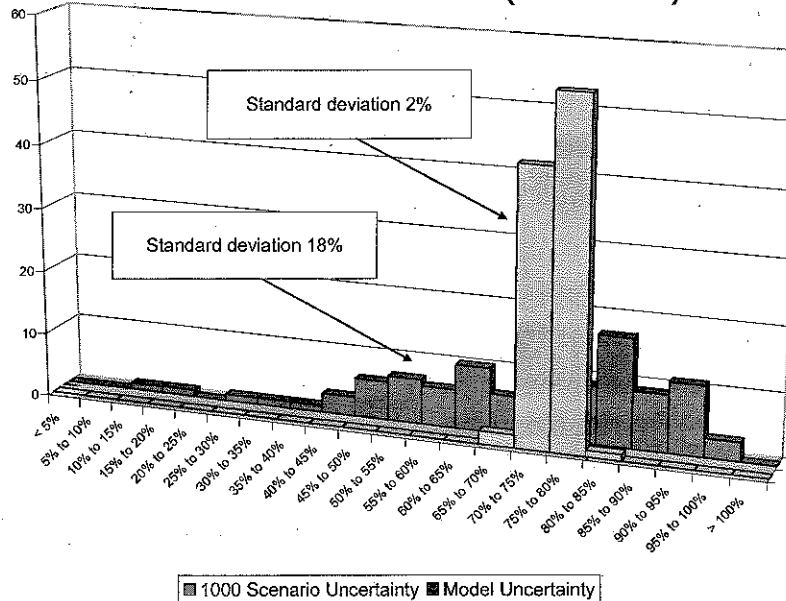
- A possible quantification of uncertainty from model selection.
 - As before, assume the market is really driven by a true scenario generator. Further assume that we know both the functional form (RSLN) but we do not know the calibration parameters.
- Experiment #2:
 - Run 100 real world paths from the true model (60 years of monthly returns).
 - For each path calibrate a scenario generator.
 - For each generator calculate CTE 75 (using the same seed each time).

Model Uncertainty

Distribution of results (\$ M)



Model Uncertainty Distribution of results (CTE %)



Model Uncertainty (Tentative) Conclusions

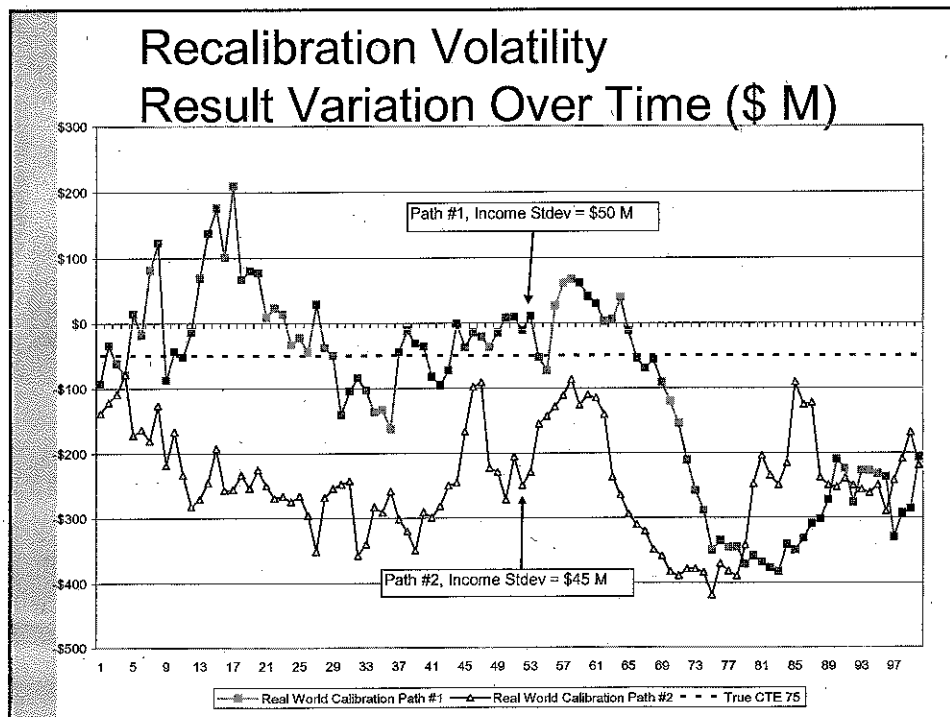
- Model uncertainty is much larger than expected, and much larger than scenario uncertainty
- OPEN QUESTIONS:
 - Do we need to focus time (and money) on reducing scenario-based uncertainty?
 - How do we deal with the larger issue of understanding what we're measuring?

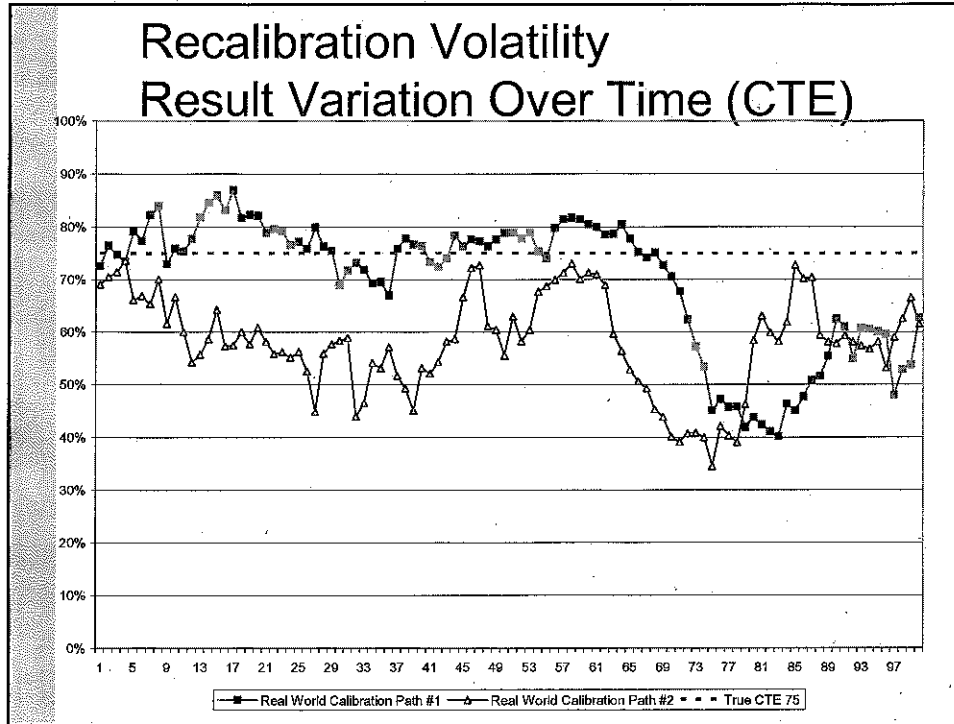
"To convert a model into a quantitative formula is to destroy its usefulness as an instrument of thought."

- John Maynard Keynes

Experiment #3 Evaluating recalibration

- Typically we periodically recalibrate our models. This experiment shows the volatility over time that emerges from recalibration.
- Experiment #3:
 - Run 1 real world path from the true model for 160 years.
 - Perform 100 running (overlapping) calibrations
 - e.g., years 1-60 followed by year 2-61, 3-62, etc.
 - Re-run the *same* liability with each of these 100 sequential calibrations
 - In reality, a year of market performance will primarily affect the NAR, and secondarily the recalibration. Here we eliminate the primary effect.





Volatility Due to Recalibration (Tentative) Conclusions

- Along a given real-world path (spurious) income volatility due to recalibration is non-negligible in size
 - Similar to arbitrarily reseeding the random number generator with 1,000 scenarios
- OPEN QUESTIONS:
 - Does it make sense to recalibrate regularly?
 - Should all market players recalibrate on a specified schedule?

Economic Scenario Generation What Have I Learned?

- The typical focus on scenario-based uncertainty is due to the fact that it is easy to measure and alleviate, not due to the fact that it is larger than other sources of uncertainty.
- We may need to rethink certain issues:
 - Are our models telling us what we think they're telling us?
 - Are we gaining or losing accuracy when we recalibrate our model?

Selected References

- Hardy, "A regime-switching model of long-term stock returns" NAAJ, 2001
 - The classic paper justifying the use of the RSLN model and describing how to calibrate parameters.
- Boyle and Windcliff, "The 1/n pension investment puzzle" NAAJ, 2004
 - This paper is not directly relevant to the subject matter above, but did suggest the methodology used in the modeling experiments.
- Cairns, "A discussion of parameter and model uncertainty in insurance" Insurance: Mathematics and Economics, 2000
 - Discusses types of model uncertainty and a Bayesian approach to addressing model uncertainty
- Manistre and Hancock, "Variance of The CTE Estimator" NAAJ, 2005
 - Provides a derivation of the formula for variance of the CTE due to scenario-based uncertainty
- Chueh, "Efficient stochastic modeling for large and consolidated insurance business" NAAJ, 2002
 - Develops the significance measure scenarios reduction technique.