Uses and Abuses of Stochastic Modeling in Disability Insurance

Boston/Hartford/Springfield Actuaries Club

Rick Leavitt, Smith Group

November 16, 2006

SMITH GROUP
DISABILITY & REINSURANCE CONSULTANTS

Stochastic Modeling

- Overview
- What is Random?
- Application to Disability Insurance
- Are Markets Normal?
- Lessons Learned

Stochastic?

Random House Unabridged Dictionary

sto   chas   tic

–adjective

of or pertaining to a process involving a randomly determined sequence of observations each of which is considered as a sample of one element from a probability distribution.

In other words: Random

From the Greek: *stokhastikos*   ("To Guess At")

Important Distinctions

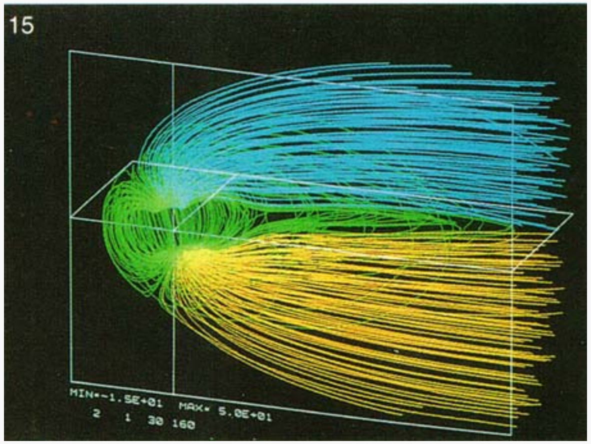
Computer Simulation

versus

Stochastic Modeling

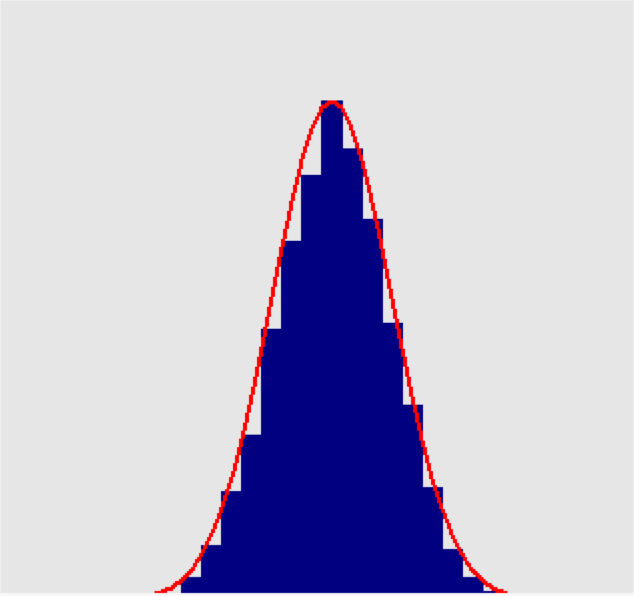
versus

Scenario Testing



What is Random?

What is Random?

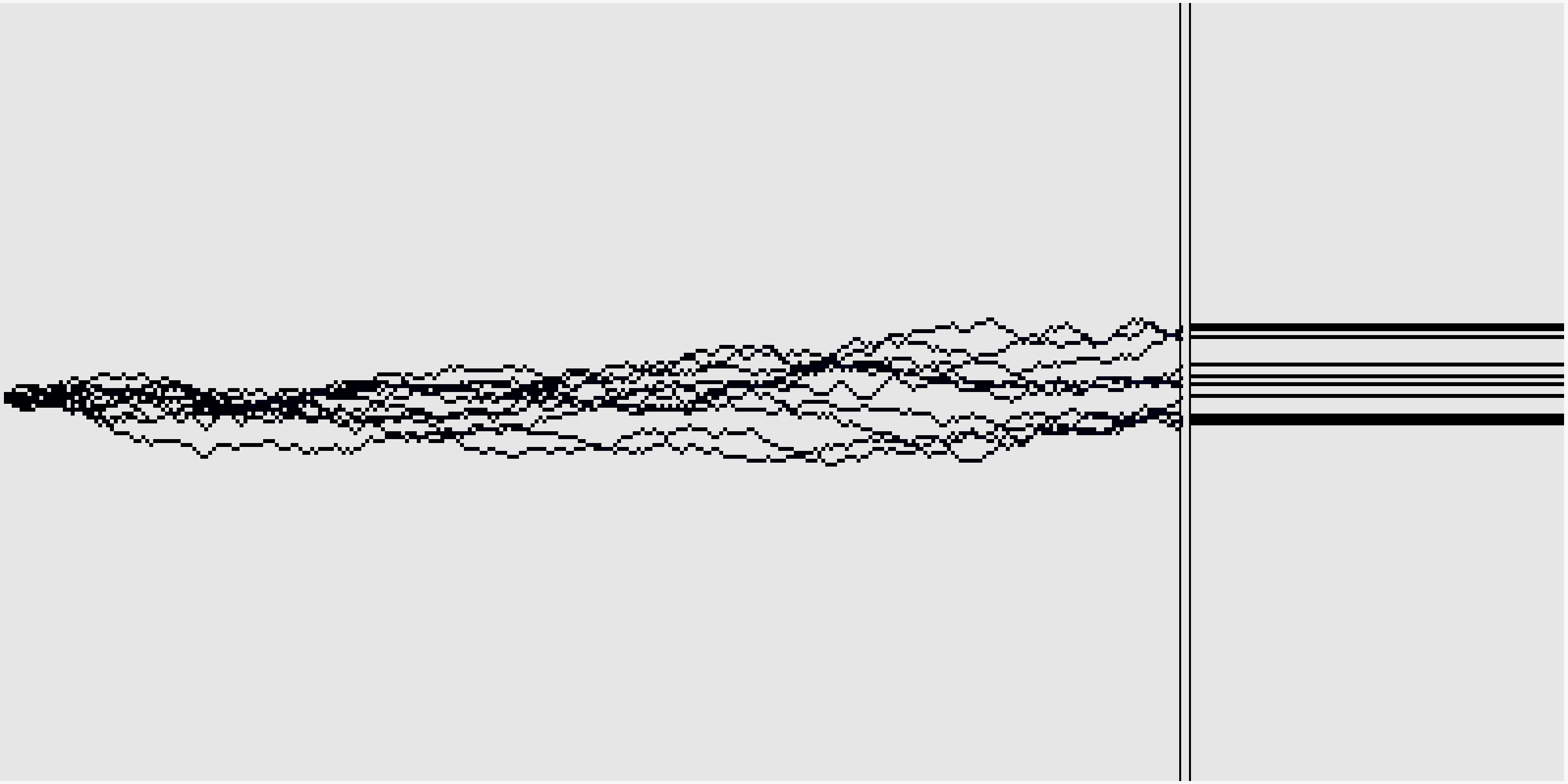


Isolated unpredictability can lead to aggregate certainty

Random Walk

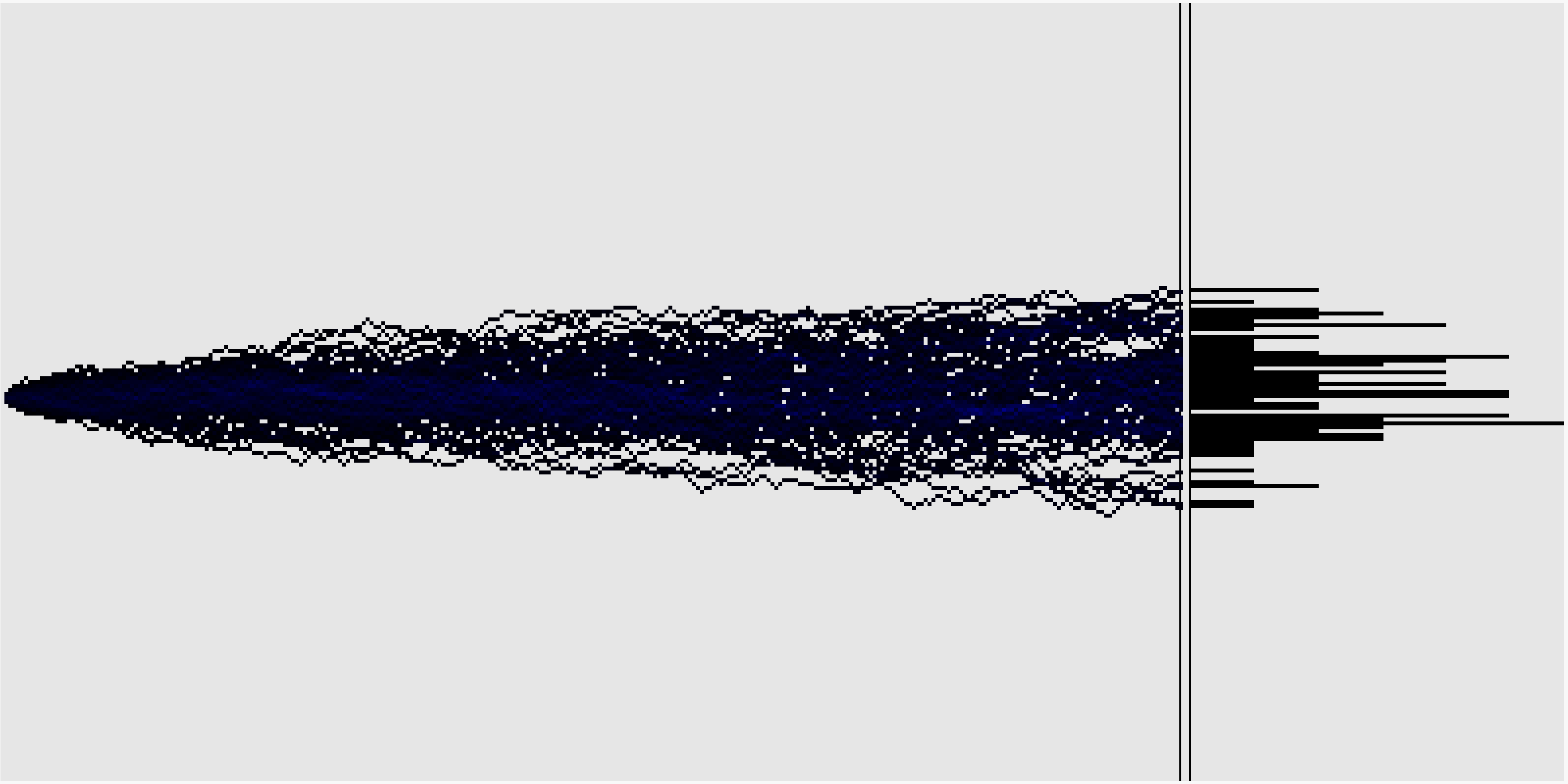
One walker: Equal chance of moving up or down

Random Walk



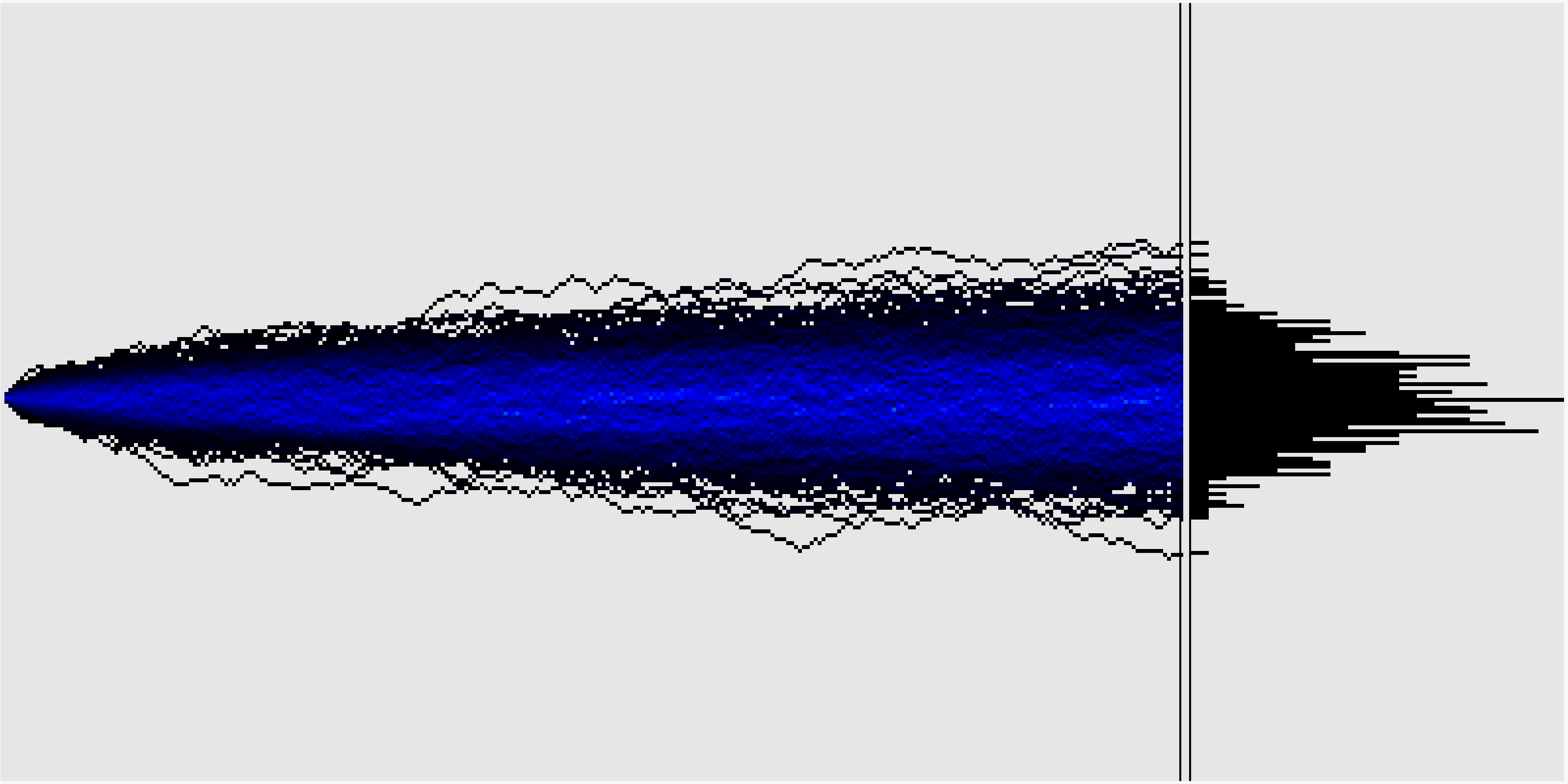
10 walkers, 600 steps each

Random Walk



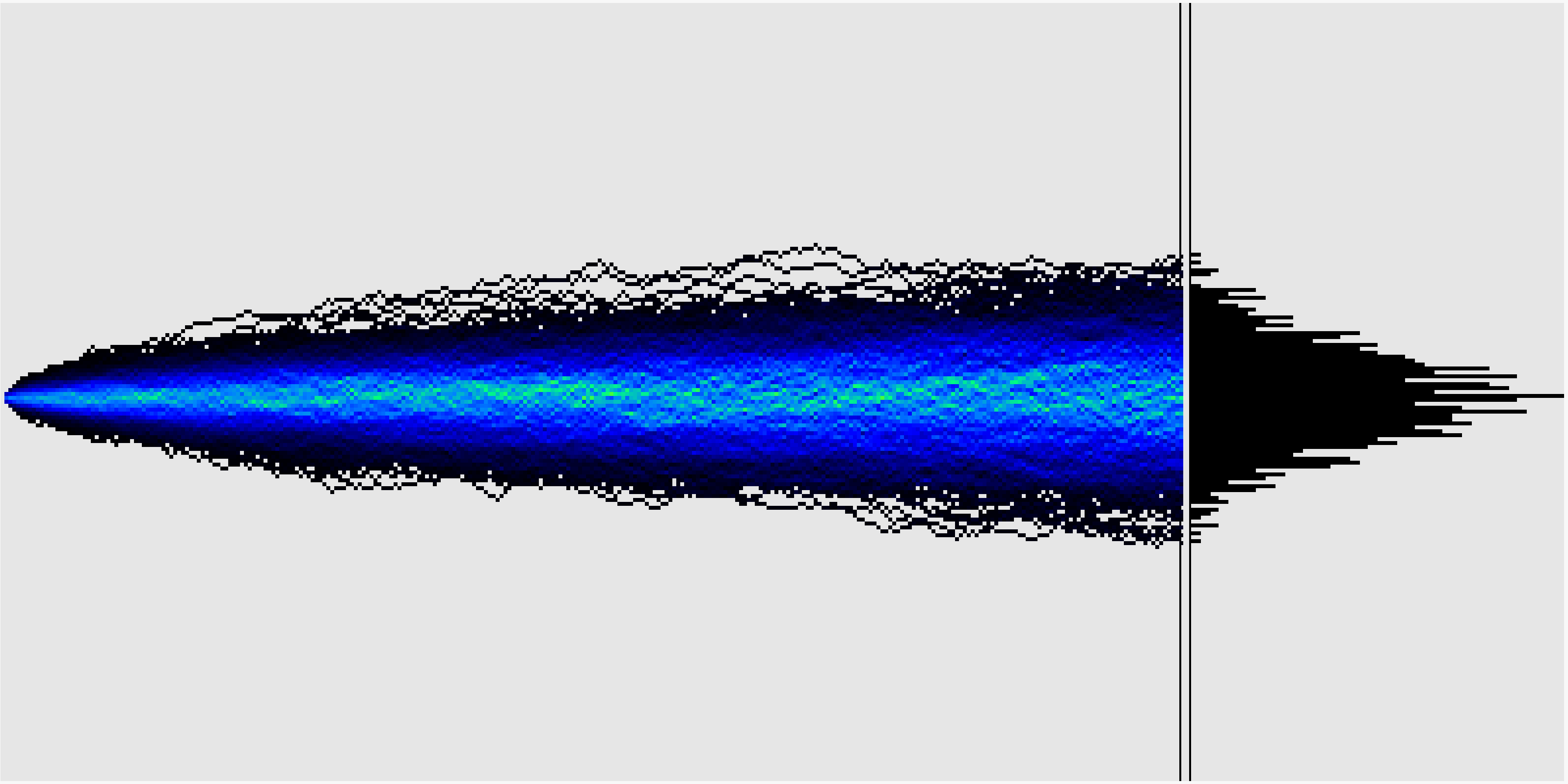
100 walkers, 600 steps each

Random Walk



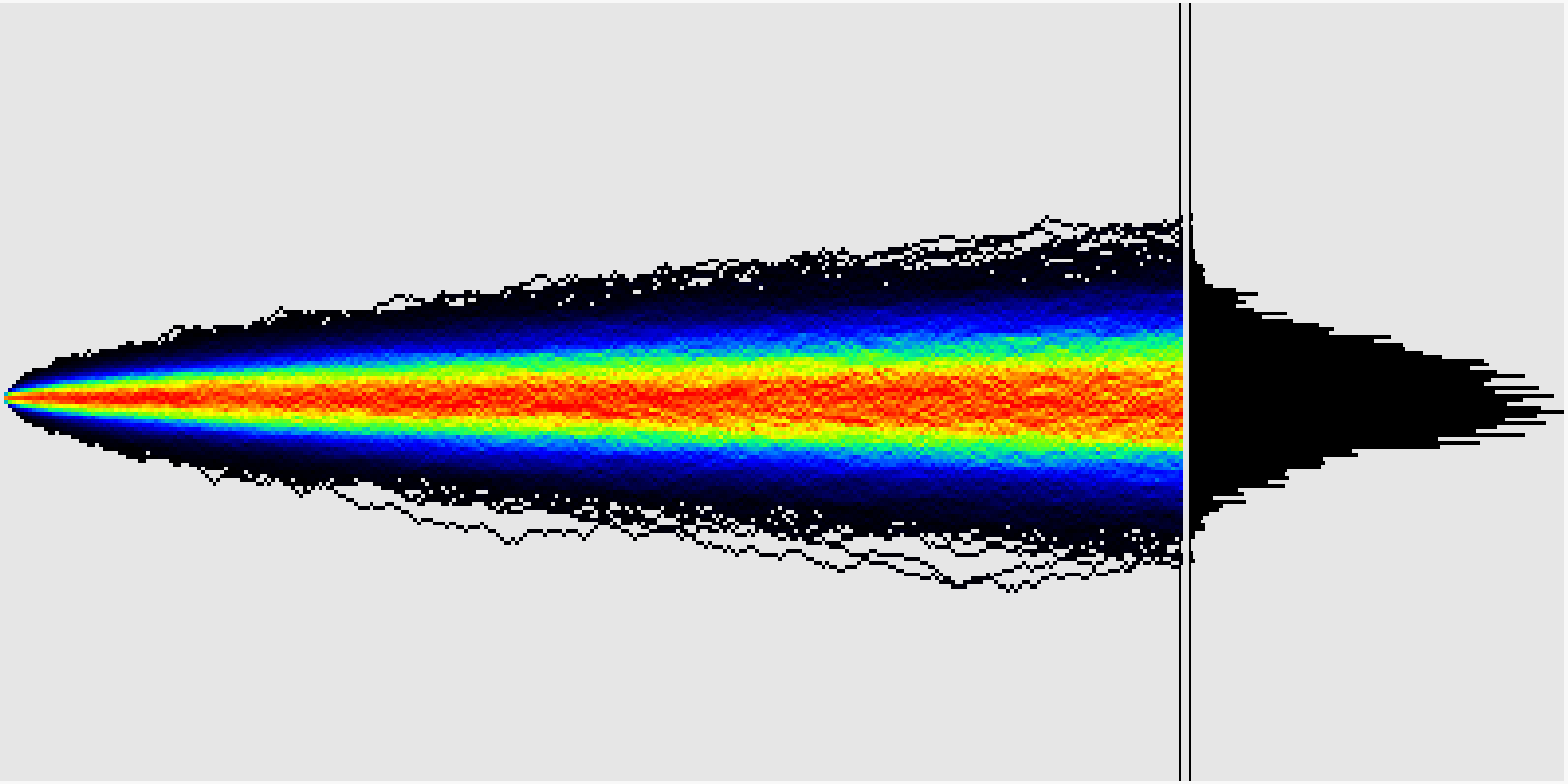
500 walkers, 600 steps each

Random Walk



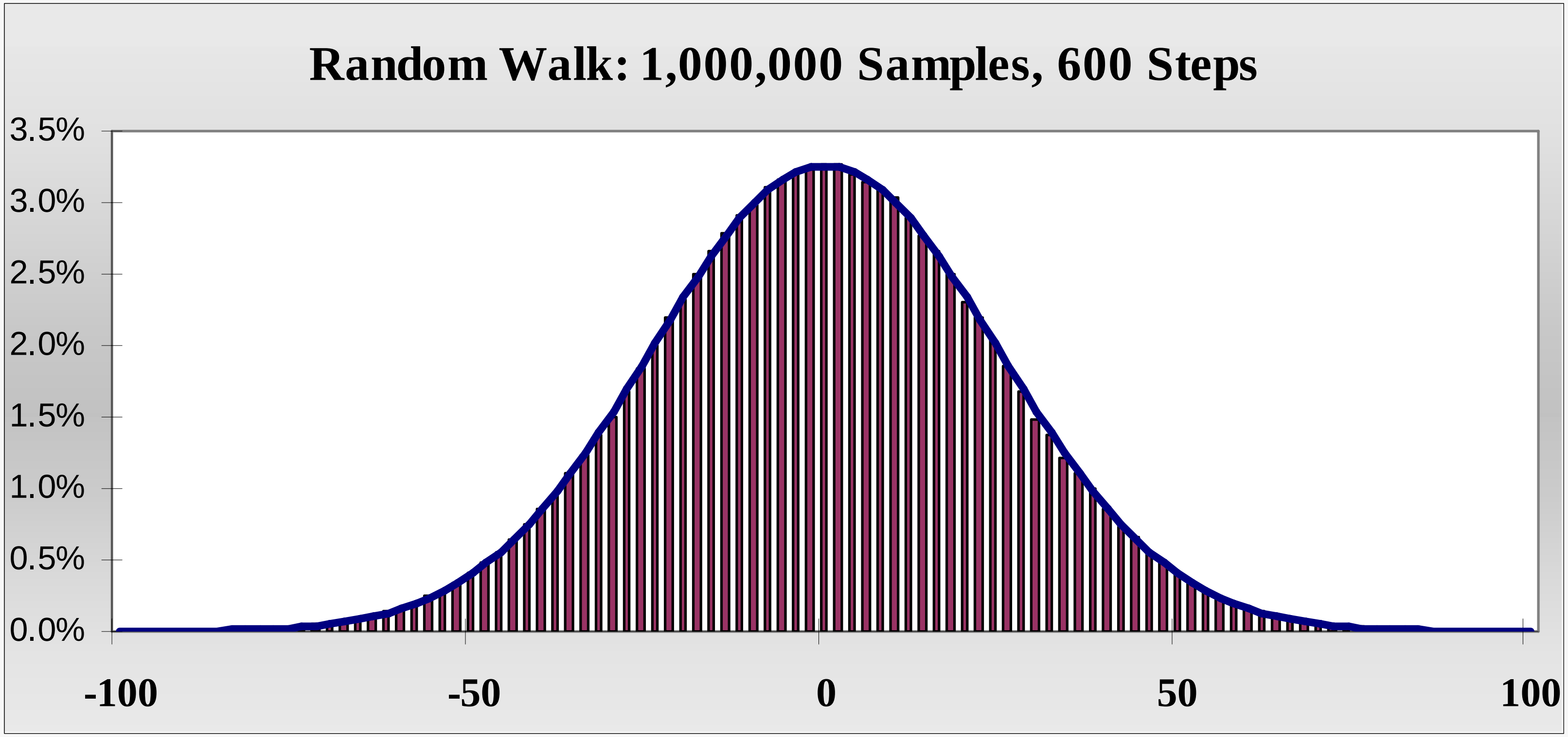
1000 walkers, 600 steps each

Random Walk



5,000 walkers, 600 steps each

Random Walk



Distribution approaches Normal with mean of 0 and standard deviation equal to the square root of the number of steps

Properties of a Normal Distribution

Mean = Median = Mode

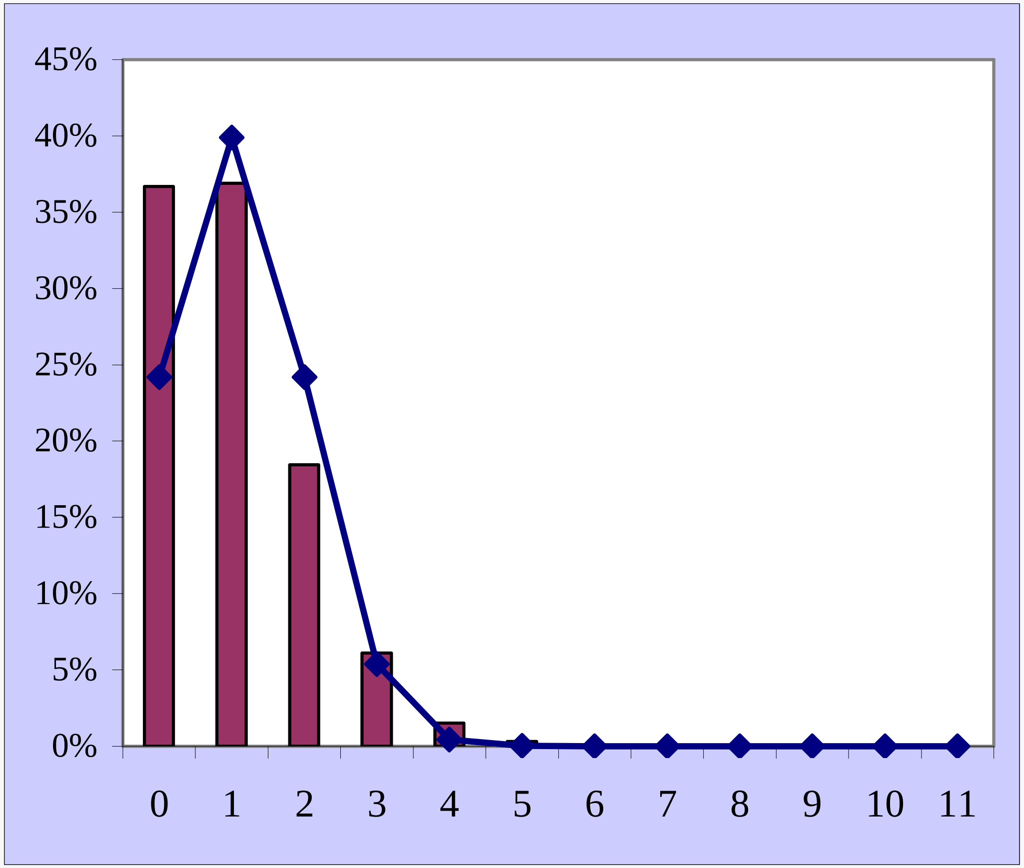
Outliers are Prohibitively Rare

Probabilities of being less than …		
Mean minus One Std Dev	15.9%	
Mean minus Two Std Dev	0.13%	
Mean minus Three Std Dev	0.0032%	
Mean minus Four Std Dev	2.87E-07	
Mean minus Five Std Dev	9.87E-10	<= One in a Billion
Mean minus Ten Std Dev	1.91E-28	<= Will never happen

Disability Incidence

Incident Rate (per 1000)	5
Number of Lives	200
Expected Claims	1

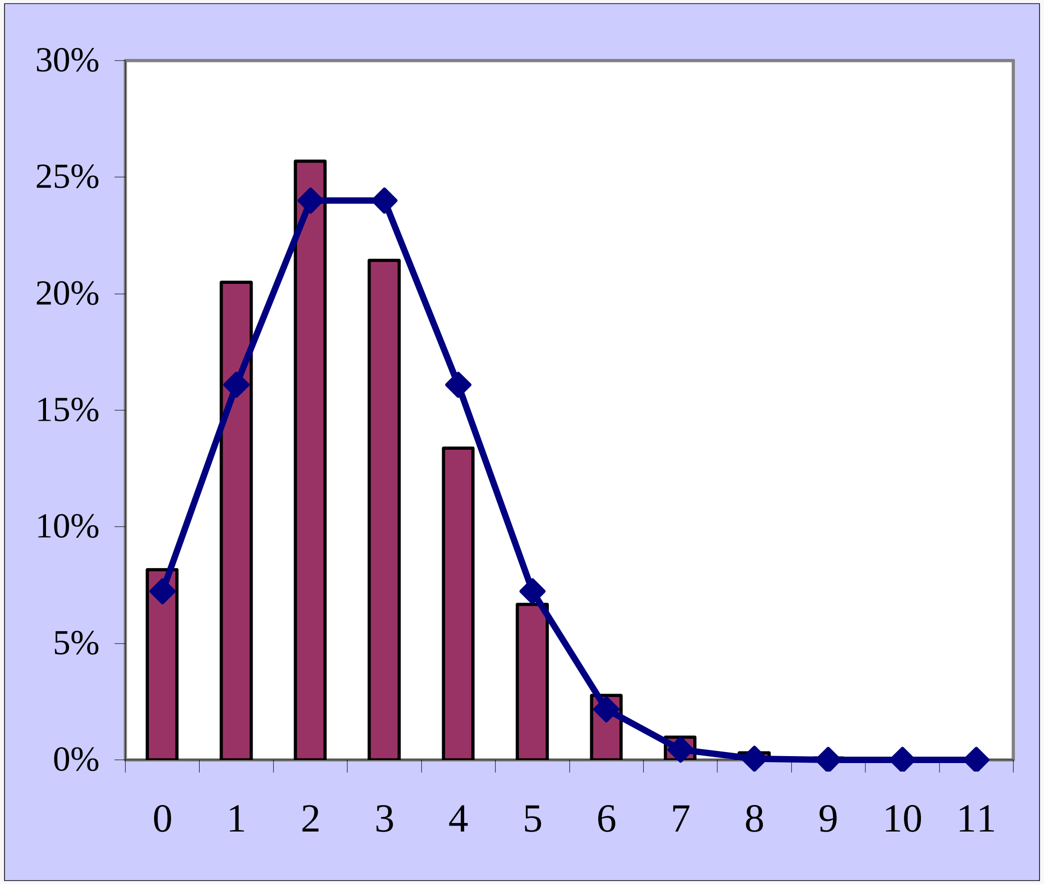
Claims	Chance	Normal
0	37%	24%
1	37%	40%
2	18%	24%
3	6%	5%
4	2%	0%
5	0%	0%
6	0%	0%
7	0%	0%
8	0%	0%
9	0%	0%
10	0%	0%
11	0%	0%



Disability Incidence

Incident Rate (per 1000)	5
Number of Lives	500
Expected Claims	2.5

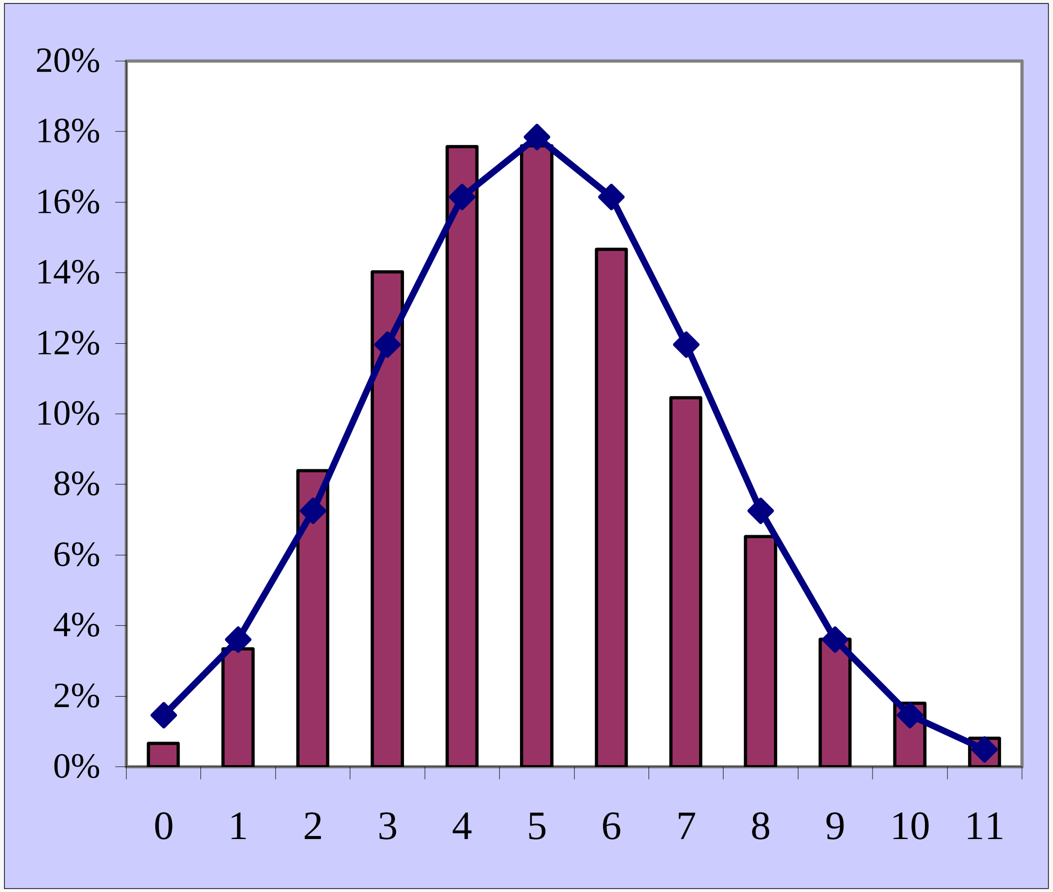
Claims	Chance	Normal
0	8%	7%
1	20%	16%
2	26%	24%
3	21%	24%
4	13%	16%
5	7%	7%
6	3%	2%
7	1%	0%
8	0%	0%
9	0%	0%
10	0%	0%
11	0%	0%



Disability Incidence

Incident Rate (per 1000)	5
Number of Lives	1000
Expected Claims	5

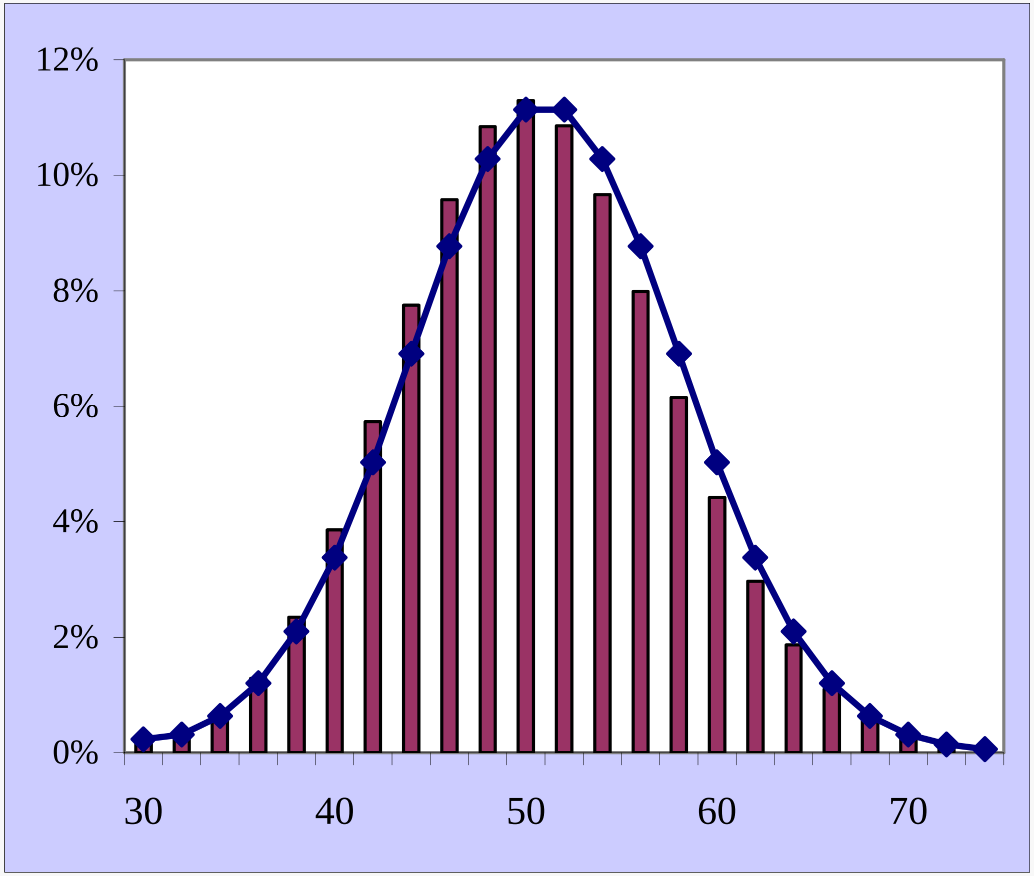
Claims	Chance	Normal
0	1%	1%
1	3%	4%
2	8%	7%
3	14%	12%
4	18%	16%
5	18%	18%
6	15%	16%
7	10%	12%
8	7%	7%
9	4%	4%
10	2%	1%
11	1%	0%



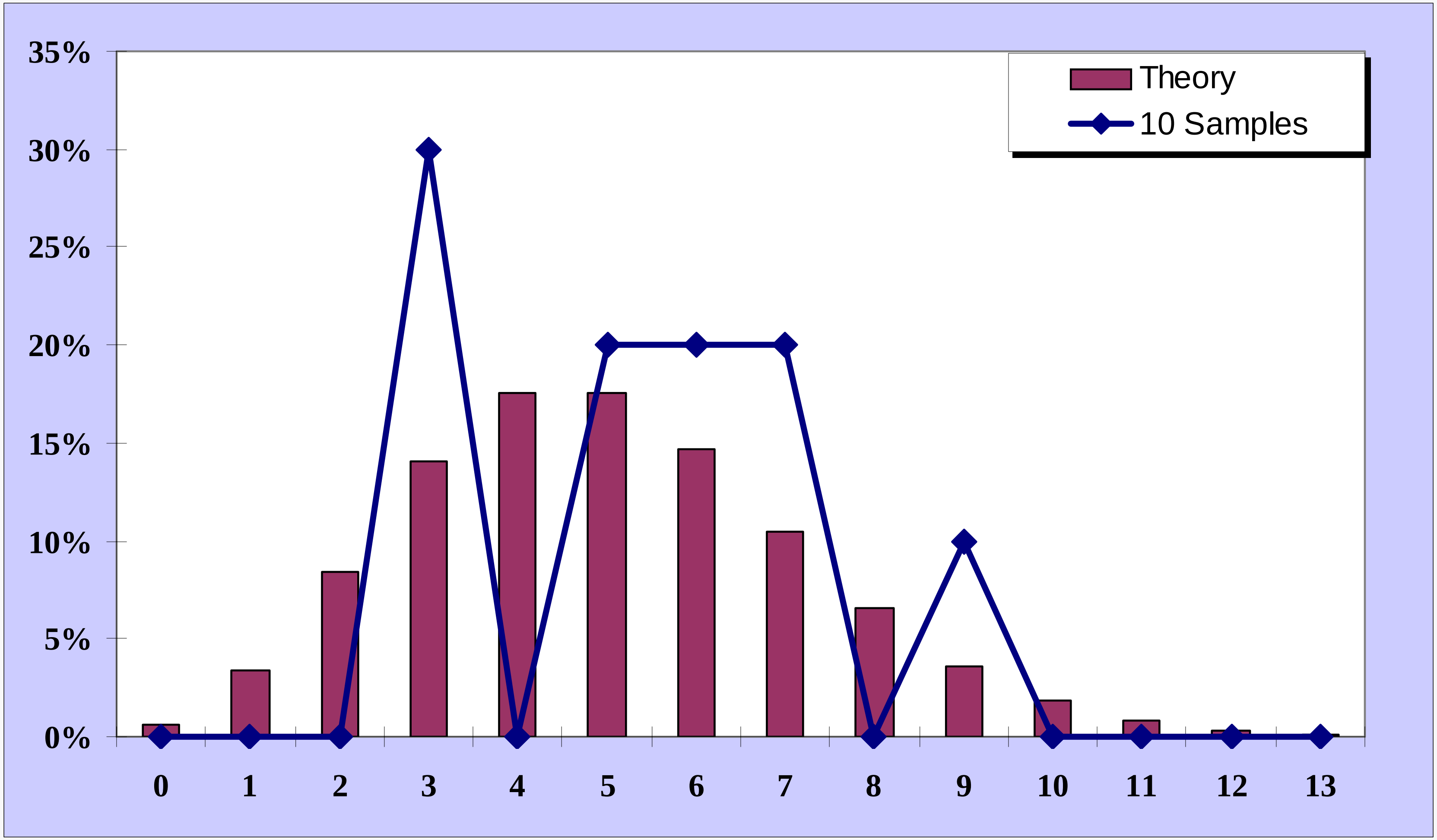
Disability Incidence

Incident Rate (per 1000)	5
Number of Lives	10000
Expected Claims	50

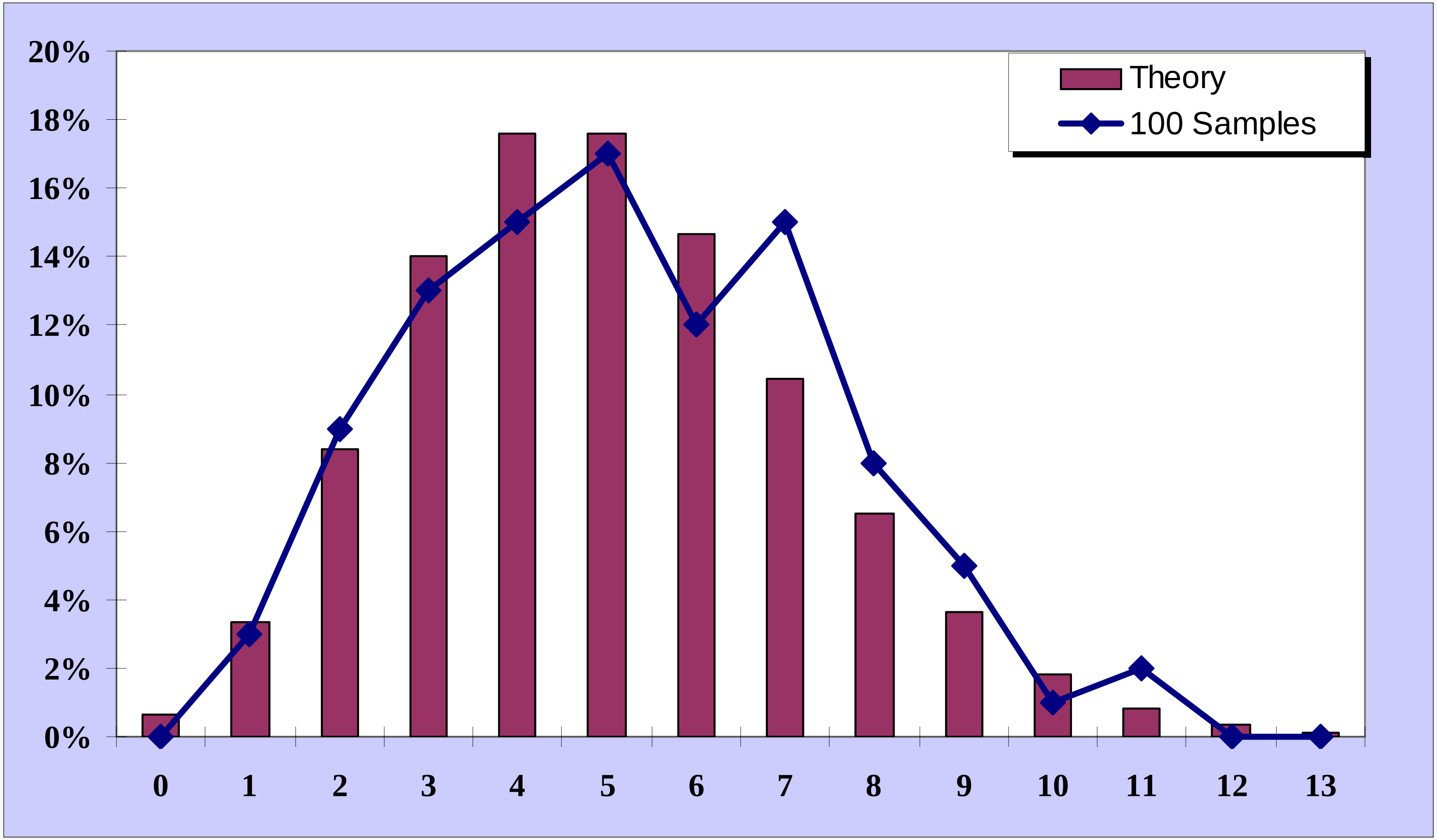
Claims	Chance	Normal
30	0.2%	0.2%
35	1.4%	1.5%
40	7.0%	6.2%
45	18.1%	16.1%
50	27.1%	26.0%
55	24.7%	26.0%
60	14.3%	16.1%
65	5.5%	6.2%
70	1.4%	1.5%
75	0.3%	0.2%



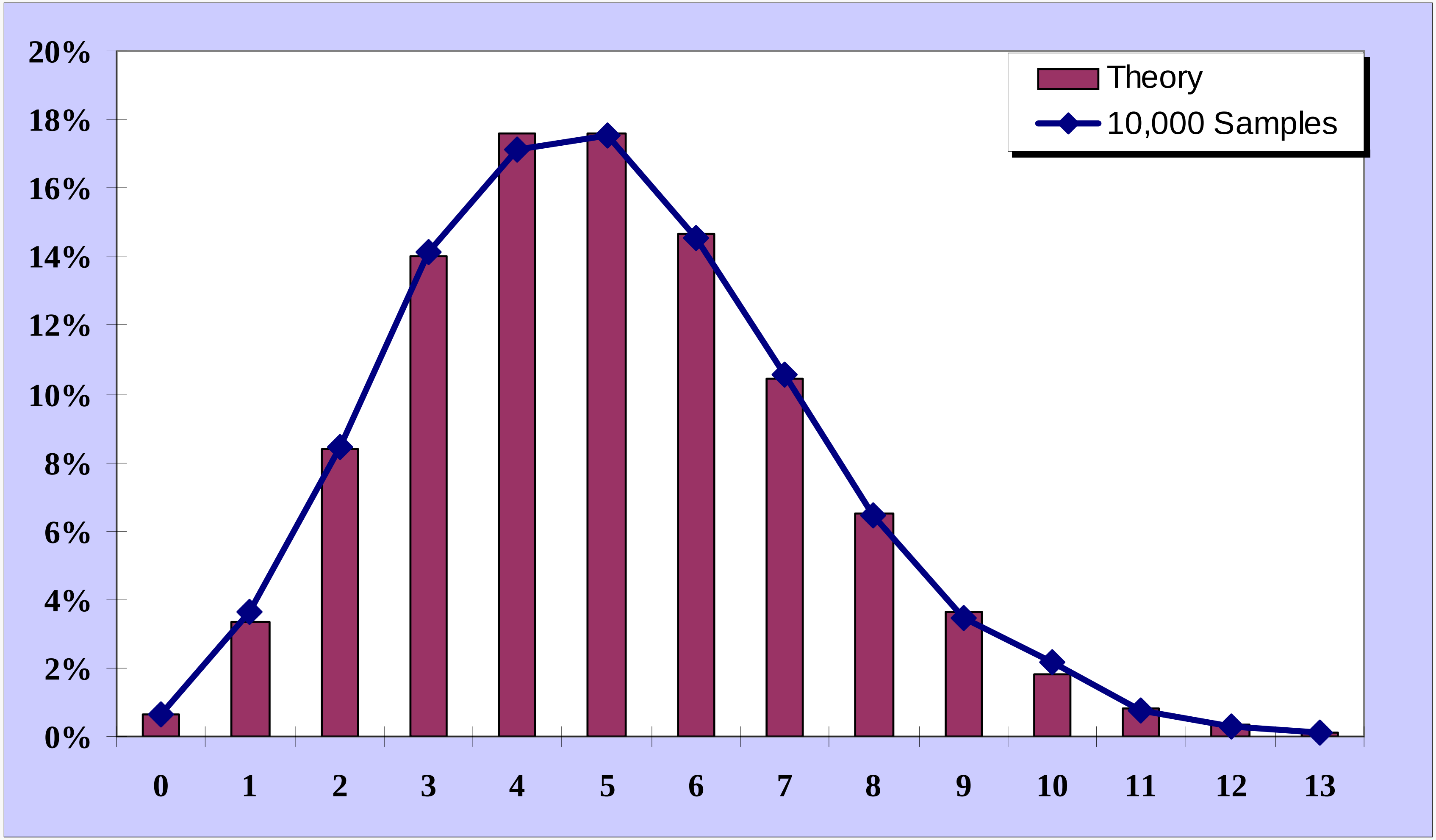
Incidence Simulation: 5 Expected Claims



Incidence Simulation: 5 Expected Claims



Incidence Simulation: 5 Expected Claims



Incidence Theory

Chance of X claims, given N lives, and Incident Rate P

Binomial Distribution

$$P(X, N, p) = \frac{N!}{(N-X)!\,X!} p^X q^{N-X}$$

Mean: *Np*

Variance: *Np(1-p)*

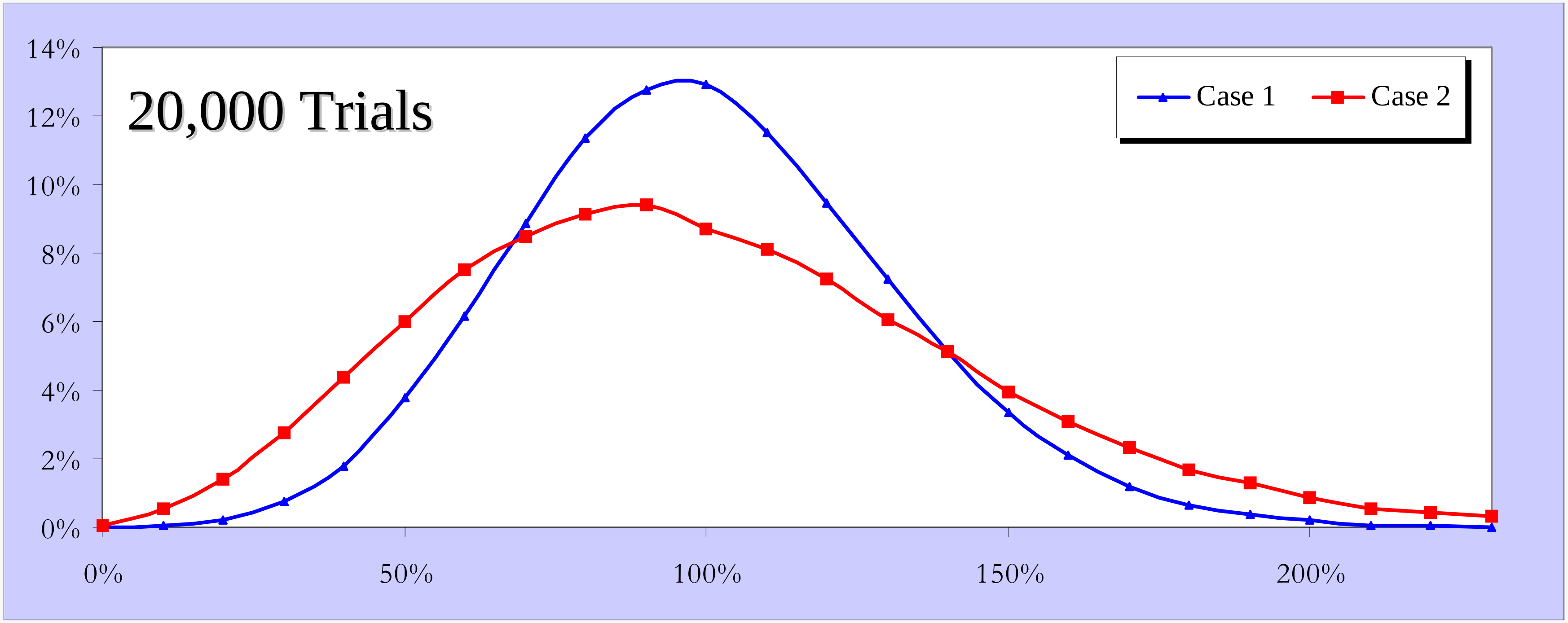
Percent Standard Deviation

$$\cong 1 / \sqrt{Np}$$

Binomial Distribution approximates Normal Distribution when

$Np(1-p) > 5$, and ($0.1 < p < 0.9$ or $\min(Np) > 10$)

Incidence plus Claim Size



Case 1: 10 expected claims: all lives have the same salary

Case 2: 10 expected claims: lives have actual salaries

Spread of Salaries does produce additional variance in the loss

If incidence and claim size are independent than the variances add.

Consider Claim Duration: (Not Normal)

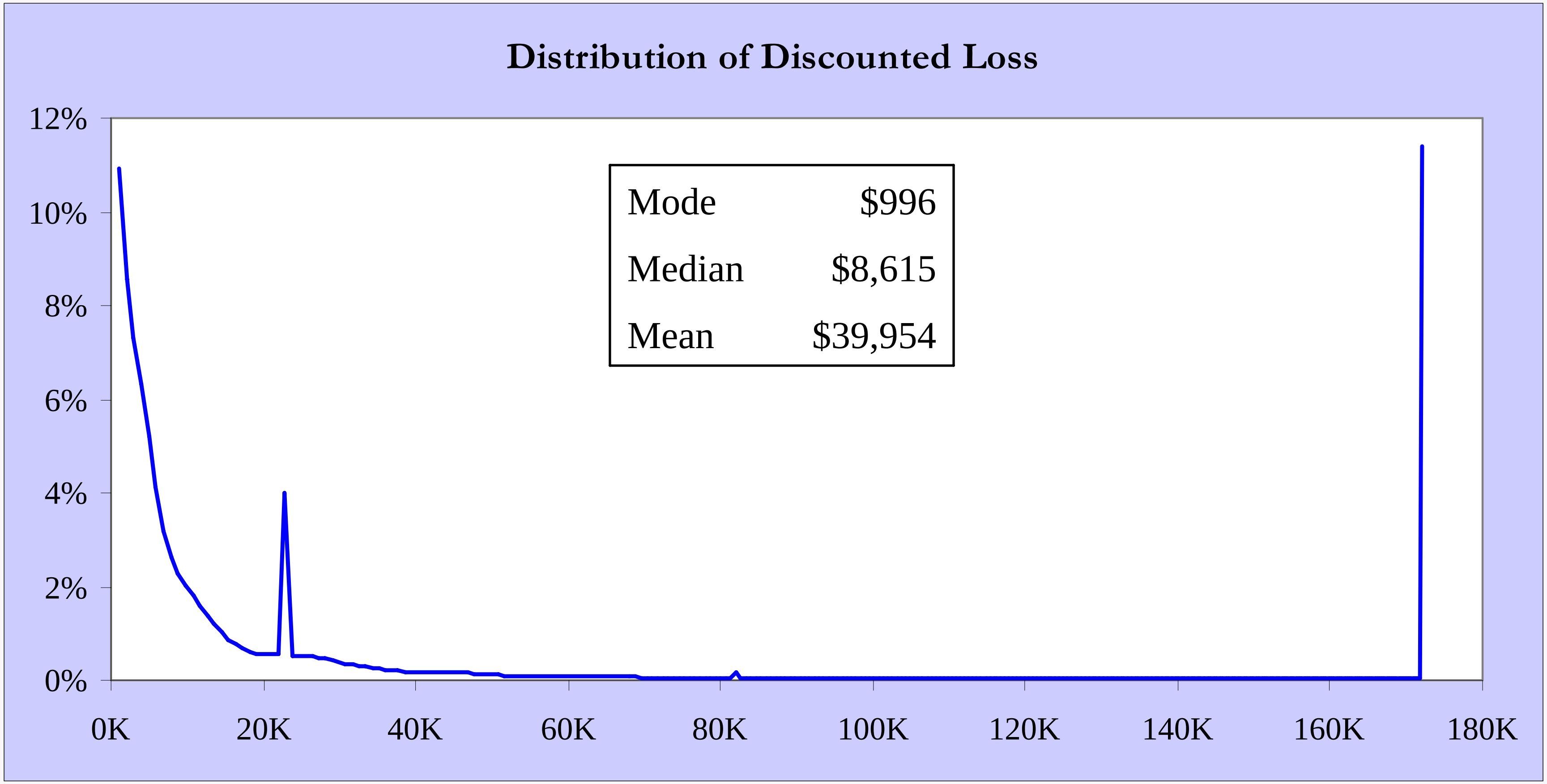


Table95a: 40 Year Old Male, 90 Day EP, $1000 Net Benefit

Stochastic Simulation: Total Claim Cost

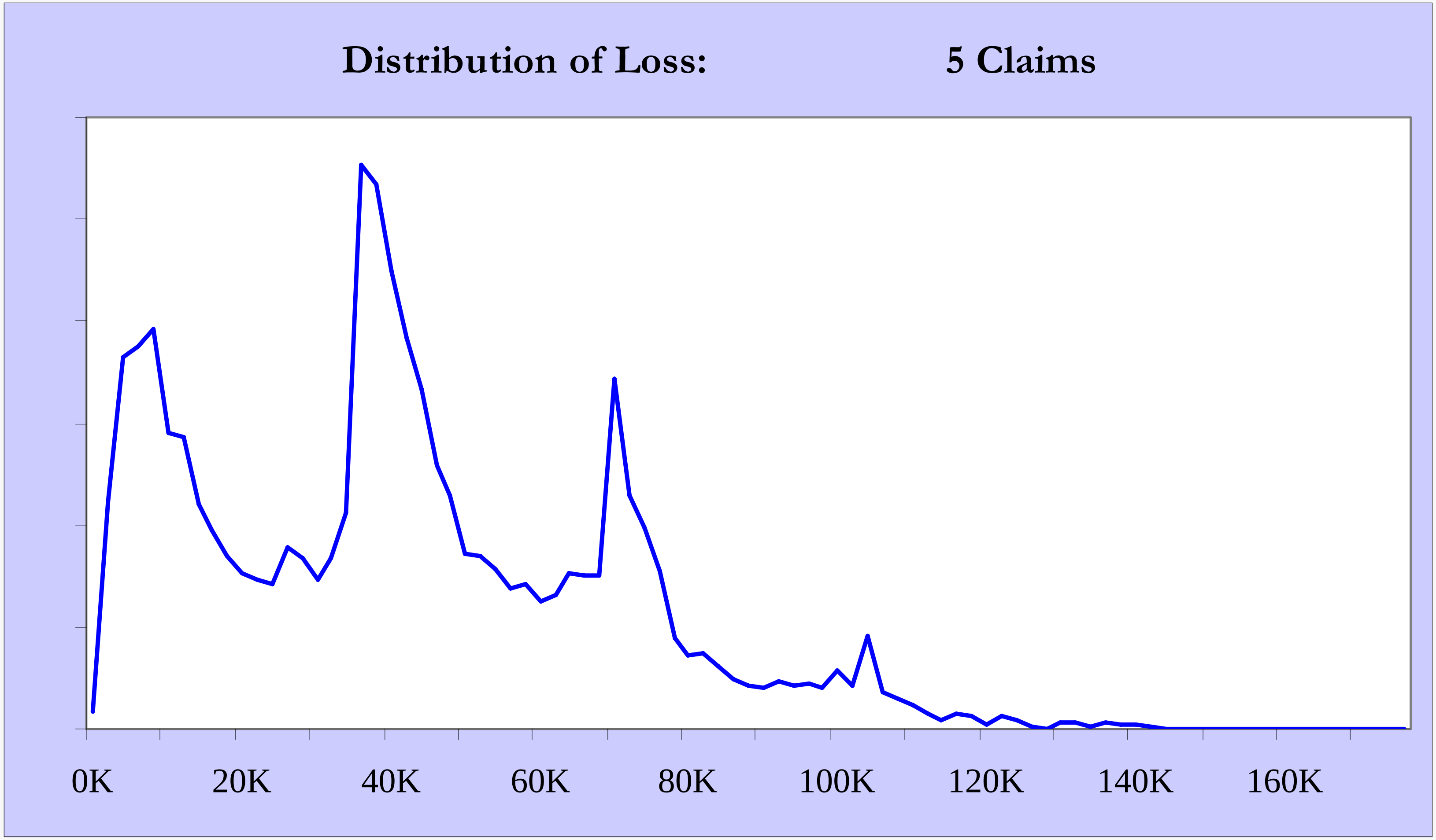


Table95a: 40 Year Old Male, 90 Day EP, $1000 Net Benefit

Stochastic Simulation: Total Claim Cost

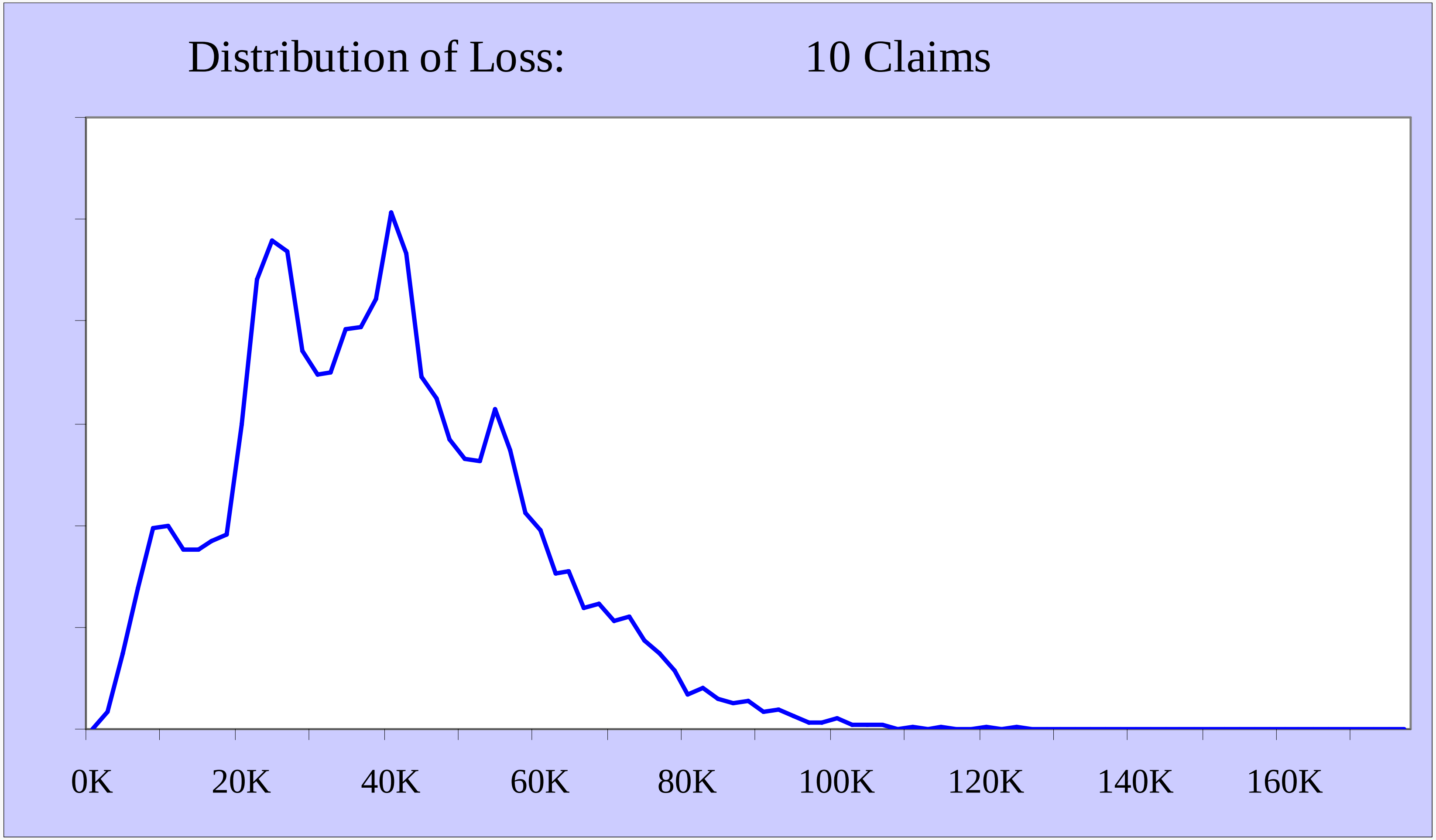


Table95a: 40 Year Old Male, 90 Day EP, $1000 Net Benefit

Stochastic Simulation: Total Claim Cost

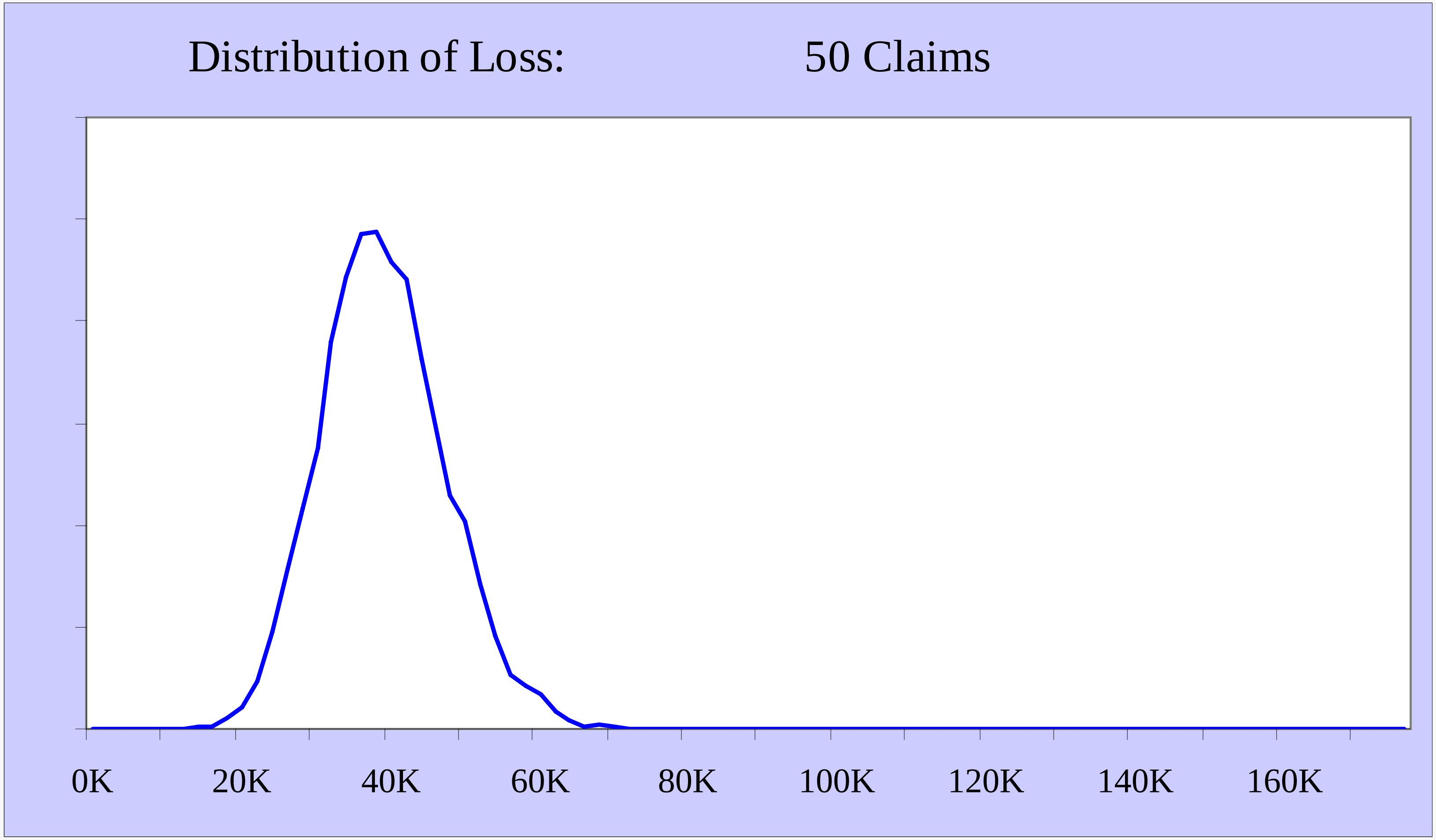
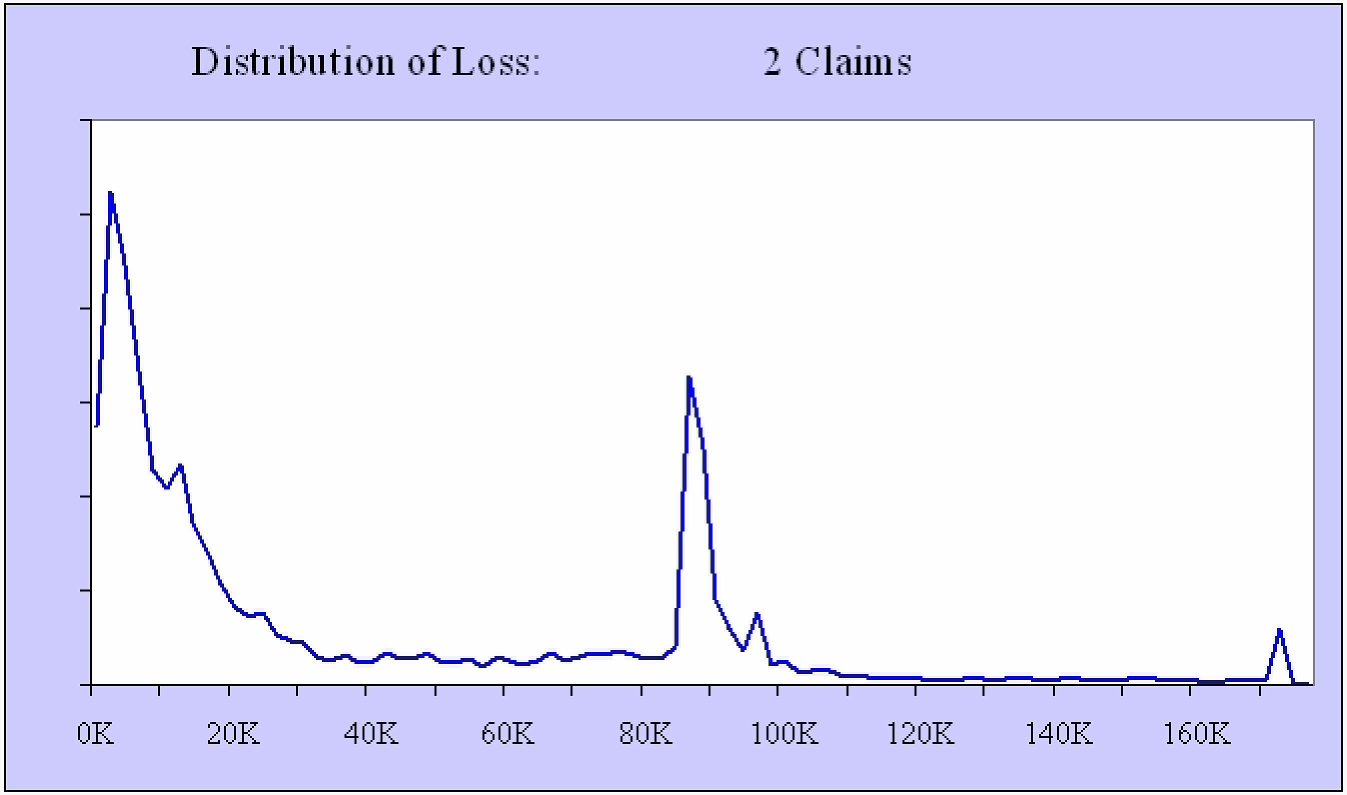


Table95a: 40 Year Old Male, 90 Day EP, $1000 Net Benefit

Stochastic Simulation: Total Claim Cost



Remember that Statistics Class?

Central Limit Theorem: The sum of independent identically distributed random variables with finite variance will tend towards a normal distribution as the number of variables increase.

*** *explains the prevalence of normal distributions* ***

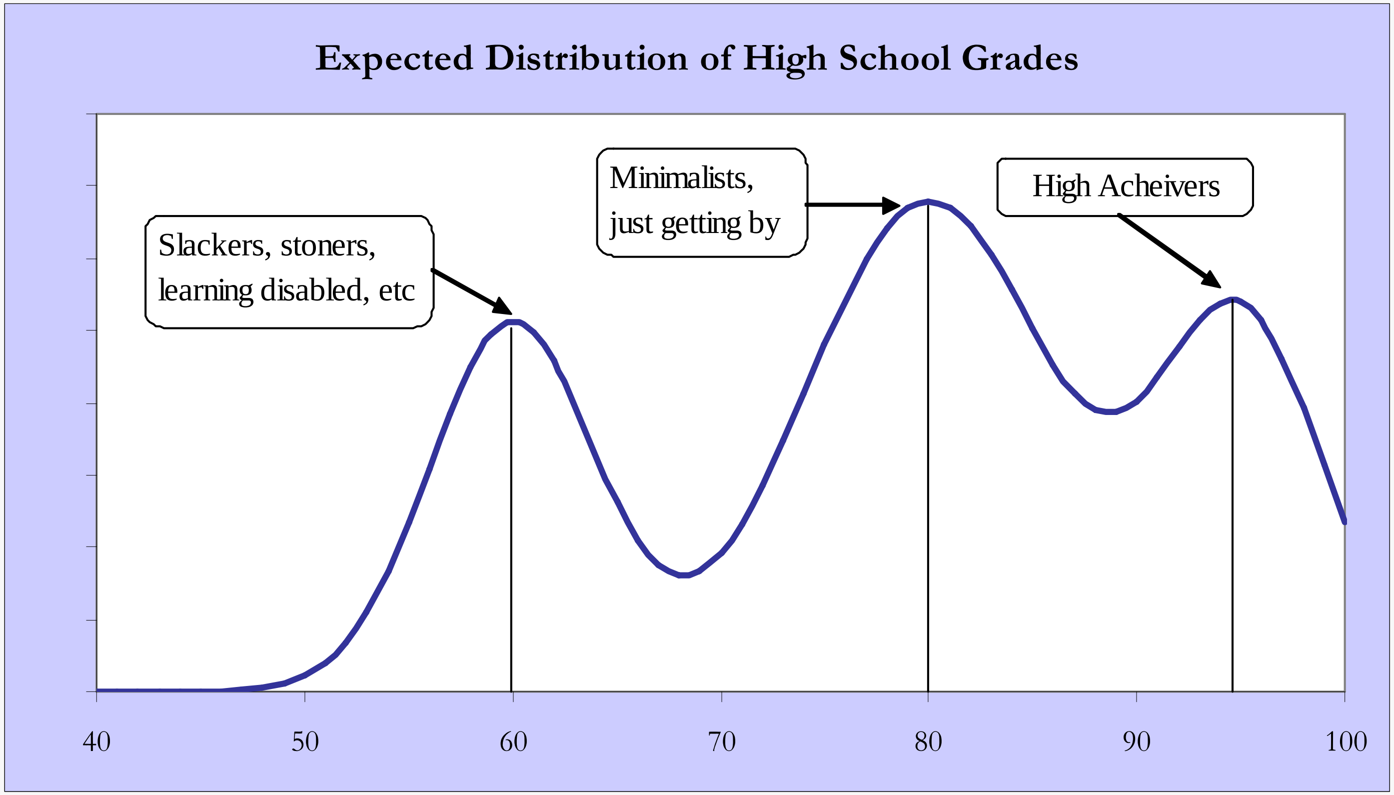
… or…

The distribution of an average tends to be *Normal*, even when the distribution from which the average is computed is decidedly non-Normal. Furthermore, this normal distribution will have the same mean as the parent distribution, *and*, variance equal to the variance of the parent divided by the sample size.

Standard Deviation varies as $1/\sqrt{N}$

Is everything Normal?

Of course not: Key is average of many independent observations with similar expectations



With three different expectations, we will expect this distribution, regardless of the number of observations, but class averages will be normally distributed

Application to Disability Insurance

For a large, and diverse block: Random variations produce expected distributions that are normal with standard deviation that varies as $X/\sqrt{N}$

•*X* depends on exposure details but varies between 1.2 and 1.5

•Simple Stochastic Simulation adds little new information

For a $500 million dollar block this translates into expected deviation of 2.5% per quarter, or 1.25% year over year.

but …

…Observed variations exceed this by at least a factor of 3 to 5

Stochastic Simulation in Disability Insurance

Let's Summarize …

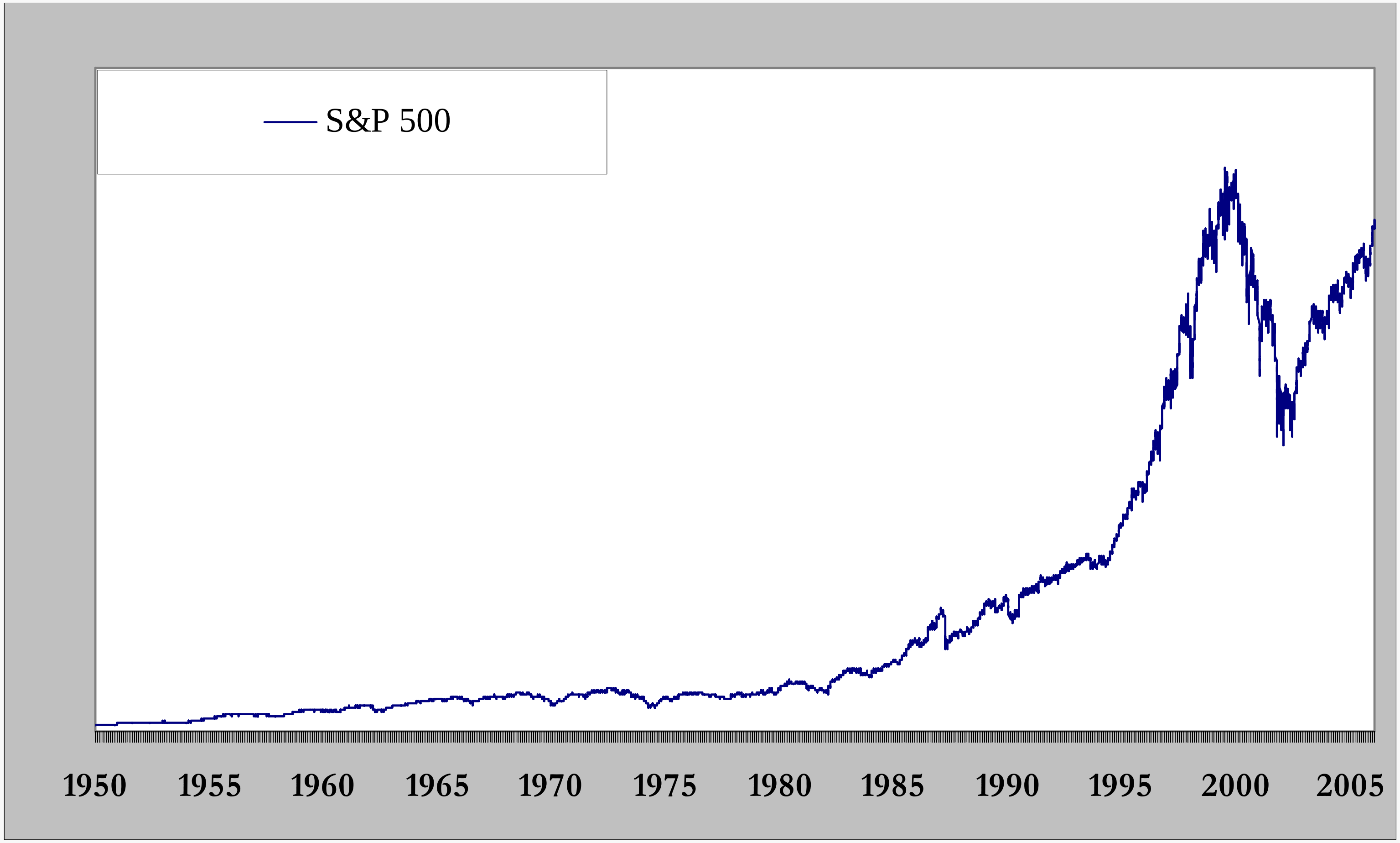
1. Results can be determined analytically without use of the stochastic model

2. Results are incorrect (inconsistent with reality)

What went wrong???

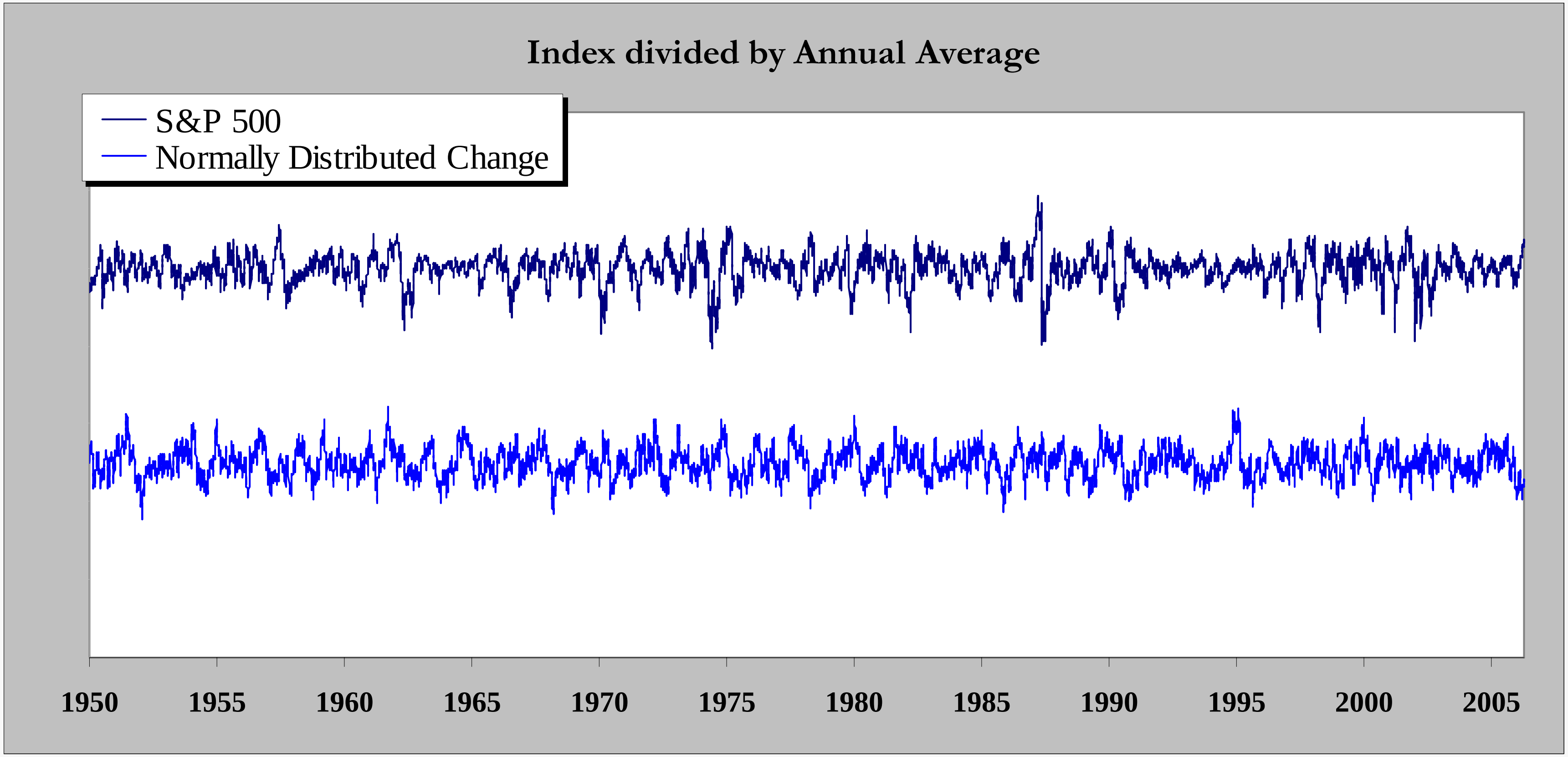
Obviously there are additional sources of variance
… but there may be something else going on.

Mild versus Severe Randomness



Take a look at other Random Processes

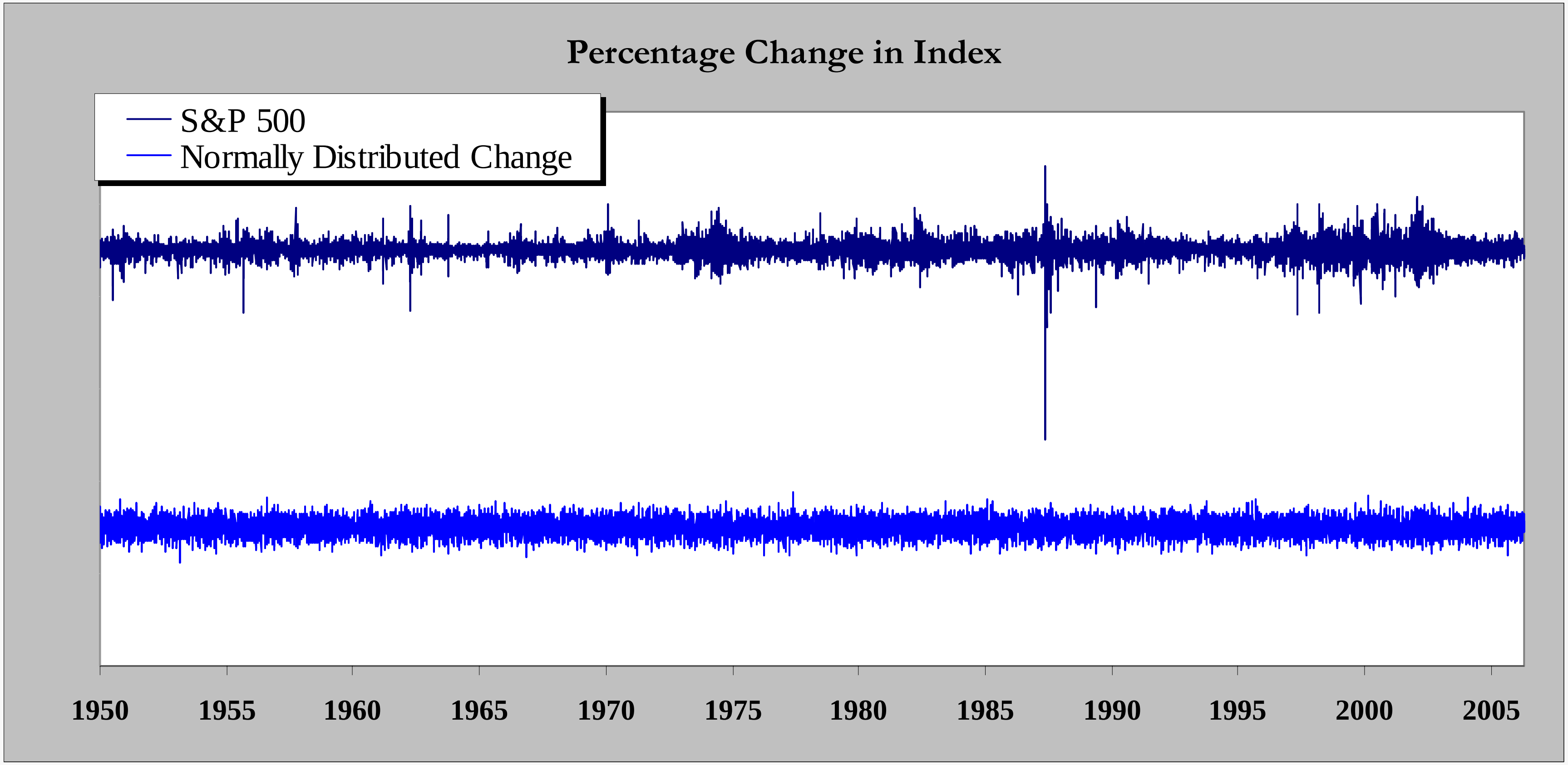
Mild versus Severe Randomness



Is random walk approach appropriate?

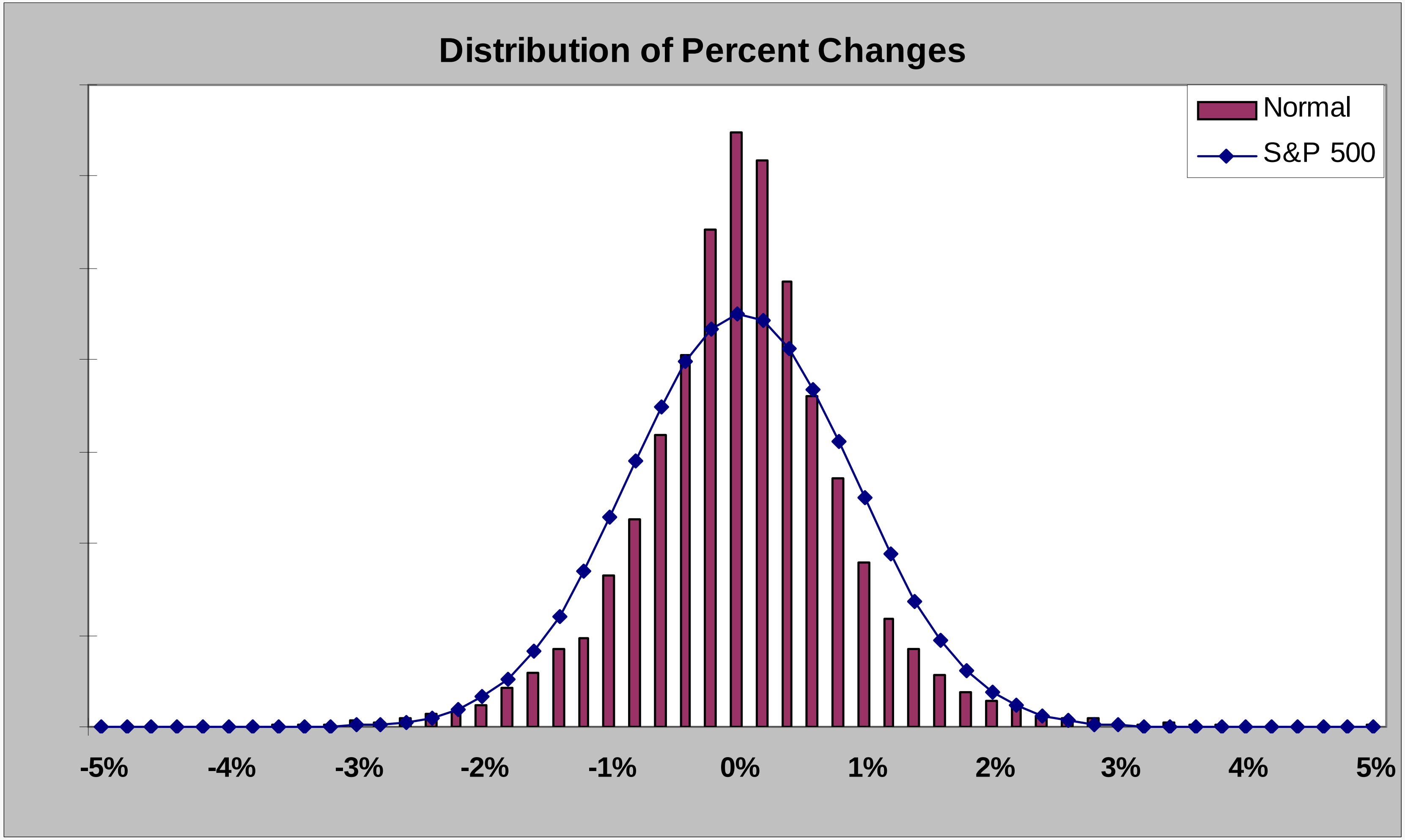
This is the assumption underlying Black-Scholes option pricing, Capital Asset Pricing Model, etc

Mild versus Severe Randomness

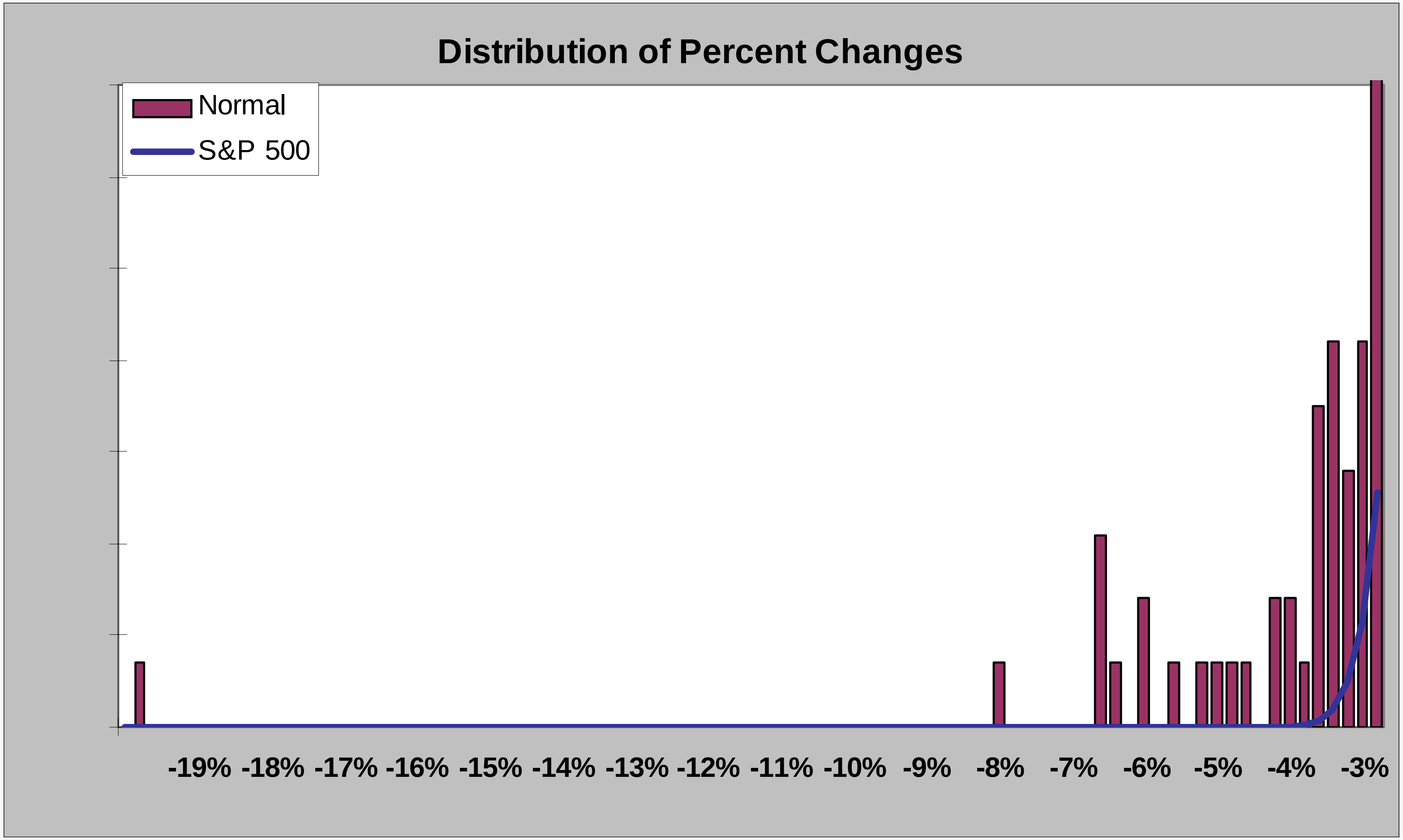


S&P Index shows higher frequency of big changes

Mild versus Severe Randomness



Mild versus Severe Randomness



Stochastic Modeling using Normal Randomness does not predict the Outliers

Mild versus Severe Randomness

S&P 500: The Biggest daily Changes

10 Worst Days

Date	Change	Odds	
10/20/87	-20.4%	5.31E-118	<== Will Never Happen
10/27/87	-8.3%	7.41E-21	
10/28/97	-6.9%	6.09E-15	
9/1/98	-6.8%	6.09E-15	
1/11/88	-6.8%	6.09E-15	
9/26/55	-6.6%	3.48E-14	
5/29/62	-6.2%	9.82E-13	
10/16/89	-6.1%	9.82E-13	
4/17/00	-5.8%	2.27E-11	
6/26/50	-5.4%	4.28E-10	<== 4 in 10 Billion

Stochastic Modeling using Normal Randomness
does not predict the Outliers

Mild versus Severe Randomness

S&P 500: The Biggest daily Changes

10 Best Days			
Date	Change	Odds	
10/30/87	4.9%	6.63E-09	6 out of 1 Billion
1/4/01	5.0%	6.63E-09	
9/9/98	5.1%	6.63E-09	
10/29/97	5.1%	1.73E-09	
10/21/87	5.2%	1.73E-09	
10/10/74	5.3%	4.28E-10	
7/30/02	5.4%	4.28E-10	
7/25/02	5.7%	2.27E-11	
5/31/62	5.9%	4.84E-12	
10/22/87	9.0%	1.02E-24	<== Will Never Happen

Stochastic Modeling using Normal Randomness
does not predict the Outliers

Mild versus Severe Randomness

In real world markets, “fat tail” experience is common.

1. S&P 500
2. NASDAQ
3. Bond Prices
4. Commodity Prices
5. Currency Markets

All show more volatile extremes than simple stochastic modeling would predict… leading to underestimating the odds of catastrophic events

What about Disability Insurance ??

What is wrong with the Random Walk?

Central Limit Theorem: The sum of **independent** identically distributed random variables with finite variance will tend towards a normal distribution as the number of variables increase.

Market Price changes are not independent.
Correlation across time: Feedback (Hysteresis)

Generally…

Negative Feedback = Stable, self-correcting behavior

Positive Feedback = Unstable, vicious cycles, death spiral, etc

Can produce wildly unpredictable behavior (chaotic)

Variability in Disability Insurance

Does Disability Insurance have a feedback mechanism?

Experience itself may, but only minimally

Insurance company process definitely does, but tends to be self correcting.

External Influences

1. Economic Cycles (global, and by region, industry, etc)
2. Underwriting Cycles (process change, reaction to sales, market dynamics)
3. Other Process Change (changes in benefits process, workloads, etc)
4. Other external factors (emergence of new disabilities, changes in work ethic or work environment, changes in Social Security practice, etc)

Lessons on Stochastic Modeling

Useful as a tool for understanding dynamics, but short on predictive power

Least useful for predicting catastrophic events (where it is most needed).

Simple application ***will*** underestimate volatility

Very useful for gaining hands-on experience and familiarity with statistical behavior

Stochastic Modeling Paradox: Can only predict qualitative behavior, but expectations of qualitative behavior are used to decide the reasonability of the model.