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Pricing Living and Death Benefits for Variable Annuities

Hartford Actuarial Club

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Variable Annuity Guarantees

- Guaranteed Minimum Death Benefit (GMDB)
  - Return of premium (ROP)
  - Roll-up
  - Ratchet
- Guaranteed Minimum Income Benefit (GMIB)
  - Account balance
  - Annuity factors
- Guaranteed Minimum Account Balance (GMAB)
  - Resets
- Guaranteed Minimum Withdrawal Benefit (GMWB)
  - Resets
  - For Life

Sample Pricing and Sensitivities - a Case Study

- Compare and contrast four pricing methodologies
  1. Mean discounted profits including rider, less mean discounted profits excluding rider
  2. As 1. but allowing for the cost of additional capital
     - Equivalent to “stochastic embedded value”
  3. As 1. but doing a market consistent valuation of the profits
     - Equivalent to “perfect hedging”
  4. As 3. but allowing for the cost of additional capital
- For a range of riders on a Variable Annuity
  - GMDB (ROP, Rollup, Ratchet)
  - GMIB
  - GMWB

The Variable Annuity Model

- Base model cell:
  - Male, 55 at issue
  - $50,000 invested, diversified over a mix of six funds
  - Dynamically varying surrender, annuitization, and withdrawal rates
    - Applied only to the projections with riders
    - Multiplier applied to base scale to reduce or increase rates, depending on “in the moneyness”
  - Hurdle rate, for return on capital, 8% (sensitivities @ 7%, 9%)
  - Target surplus = 200% of “new” capital requirements – i.e. RBC C3 Phase 2
- Sensitivities tested
  - Age at issue
  - Investment mix
  - Dynamic behavior
- Projection
  - 250 scenarios for 40 years

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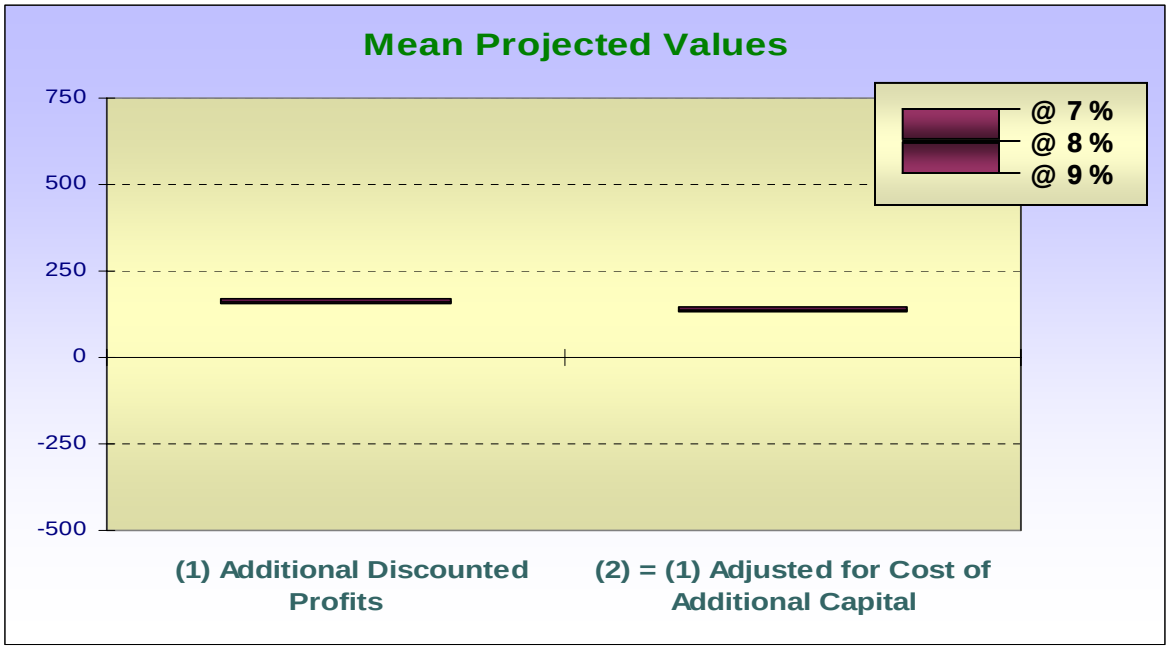
RBC C3 Phase 2 Capital Requirements

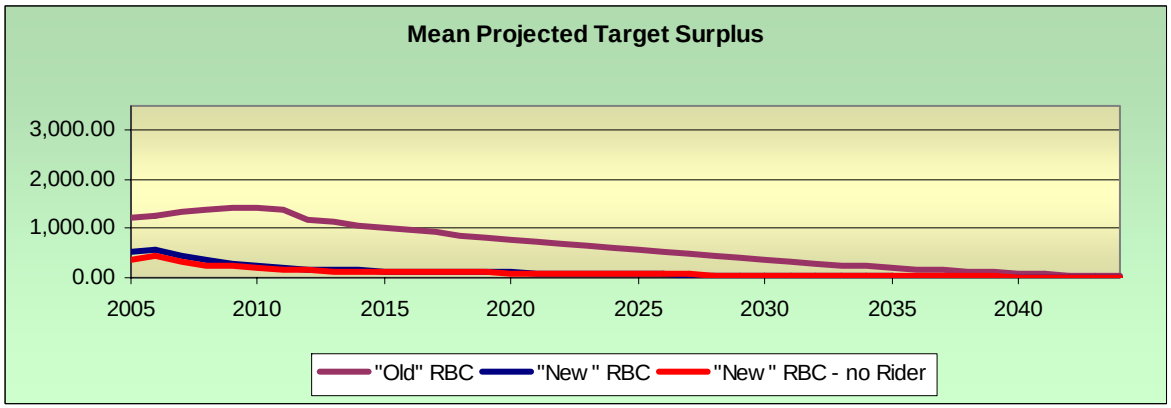
- C3 component is calculated as 90% CTE of PV of minimum surplus at each year end
  - Requires stochastic on stochastic modeling
    - Gives challenges in model coding, model performance, model verification
    - Key components
      - Using an existing actuarial projection system (Prophet)
      - Inner stochastic loop of calculations uses annual time steps
        - Care needed to distinguish between policy and company years
      - Using 250 scenarios for each inner loop (adequate for this presentation)
    - Little model optimization needed, since data is relatively small
- No allowance for aggregation across portfolio
- No allowance for a “clearly defined hedging strategy”
  - For accurate modeling this would require stochastic cubed!

Scenario Generator

- Scenarios produced using The Smith Model (TSM), Deloitte's proprietary econometric generator
  - Can produce both "real world" and "risk neutral" scenarios
  - Arbitrage free, full term structures, multiple economies
  - Optional fat tailed and skewed distributions with jumps
    - not used in this case study
  - www.thesmithmodel.com
- "Real world" calibration as at December 2004, projecting five correlated equity and bond funds, and correlated full interest rate curves
- Used deflators (more on this later) to produce market consistent valuations
  - Deflators are a natural by-product of TSM's construction
- Inner stochastic loop scenarios produced by "transposing" the base scenarios
  - Adjusts for interest rate changes between the projection start date and each valuation date
  - Volatilities assumed to be unchanged

Starting Point: GMDB with ROP (10bps)





- The mean discounted values at the three discount rates are almost equal
- Allowing for cost of additional capital (@2 x RBC) on new RBC basis has little effect
- Demonstrates that the rider (at this price) appears to add value, using this measure of value
- Target surplus on policy is less than on “old” RBC basis
  - Little risk in guarantee

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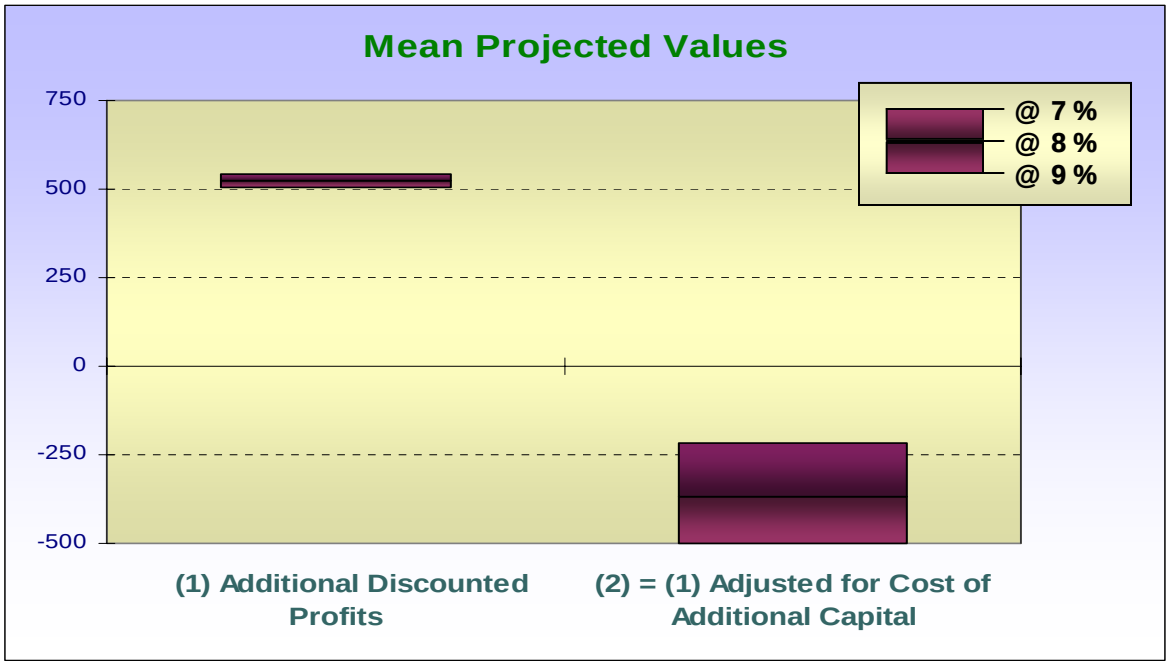
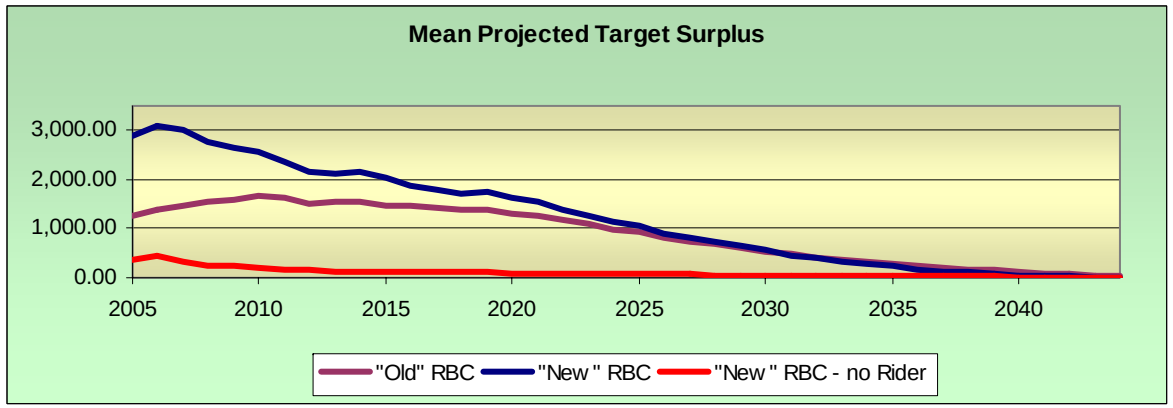
Next: GMDB with Roll Up (25bps)





- Roll up @ 5% per year
- Allowing for cost of additional capital on new RBC basis has a much greater effect
- Using this measure of value, the rider (at this price):
  - Appears to add value on old RBC basis
  - Appears to destroy value on new RBC basis
- Greater “riskiness” requires significantly greater capital
- Should one discount at a higher rate?
  - If so, what?
  - What does a Market Consistent Valuation methodology suggest?

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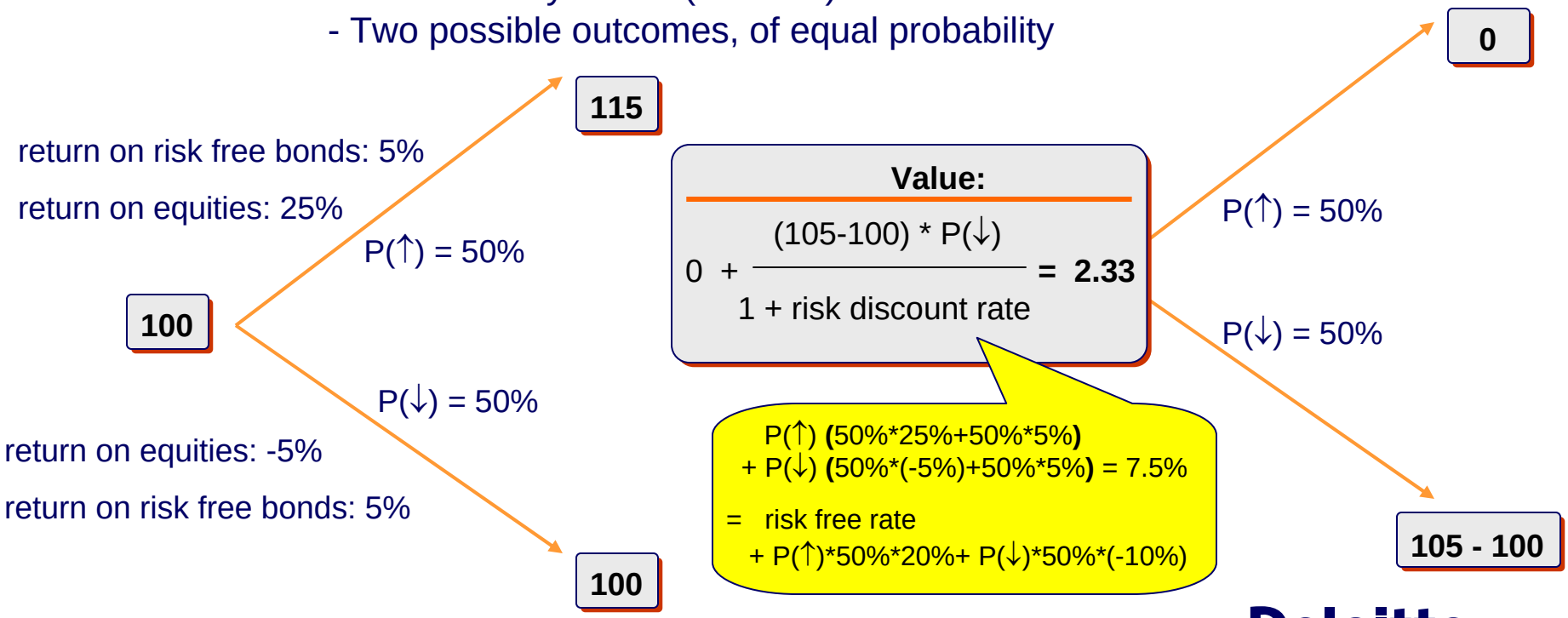
Review: Risk Adjustment Approach

- Value = expected value of outcomes using real world probabilities, discounted at the risk discount rate
- Example:

Liabilities - Premium received: 100
- Guaranteed payment to policyholder in one year's time: 105

Assets - 50% equities
- 50% treasury bonds (risk free)
- Two possible outcomes, of equal probability



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Review: Risk Neutral Approach

In order to allow no arbitrage opportunities two conditions for risk neutral probabilities must be preserved:

- Normalisation:

  $P_{RN}(\uparrow) + P_{RN}(\downarrow) = 1$

- Conservation of market efficiency:

  initial portfolio value * $P_{RN}(\uparrow)$ * (1 + return($\uparrow$))
  + initial portfolio value * $P_{RN}(\downarrow)$ * (1 + return($\downarrow$))
  = initial portfolio value * (1 + risk free rate)

Yielding the risk neutral probabilities :

$$P_{RN}(\uparrow) = \frac{\text{risk free rate} - \text{return}(\downarrow)}{\text{return}(\uparrow) - \text{return}(\downarrow)} \qquad P_{RN}(\downarrow) = \frac{\text{return}(\uparrow) - \text{risk free rate}}{\text{return}(\uparrow) - \text{return}(\downarrow)}$$

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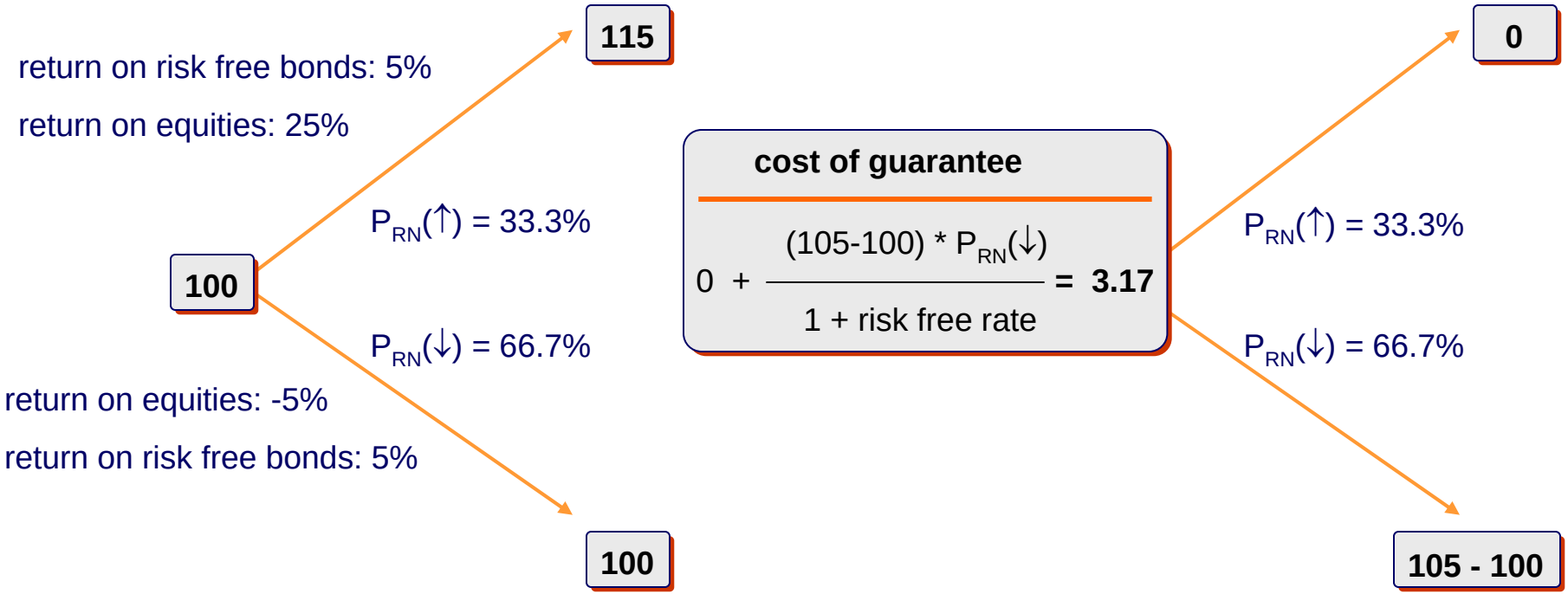
Review: Risk Neutral Valuation

- Cost of guarantee = expected value of outcomes using risk neutral probabilities, discounted at the risk free rate



Review: Risk Neutral - No Arbitrage

Two different prices for a guarantee would lead to an arbitrage opportunity:

- Buy an guarantee (option) priced with the risk adjustment approach for 2.33
- Sell it in the efficient market for the risk neutral price of 3.17

Giving:

Riskless gain of 0.84 per guarantee (option)

In efficient markets (markets with no arbitrage possibilities), risk neutral pricing persists.

Introduction to Deflators

Some economic generators (including TSM) can generate (state-price) deflators

- Effectively stochastic risk discount factors
- Vary over time and by simulation
- Can be used to value any cash flow

Using risk neutral probabilities ($P_{RN)}$ and discounting with risk free rates corresponds to using real world probabilities (P) and discounting with deflators (D):

$$D(\uparrow) * P(\uparrow) = \frac{1}{1 + \text{risk free rate}} * P_{RN}(\uparrow)$$

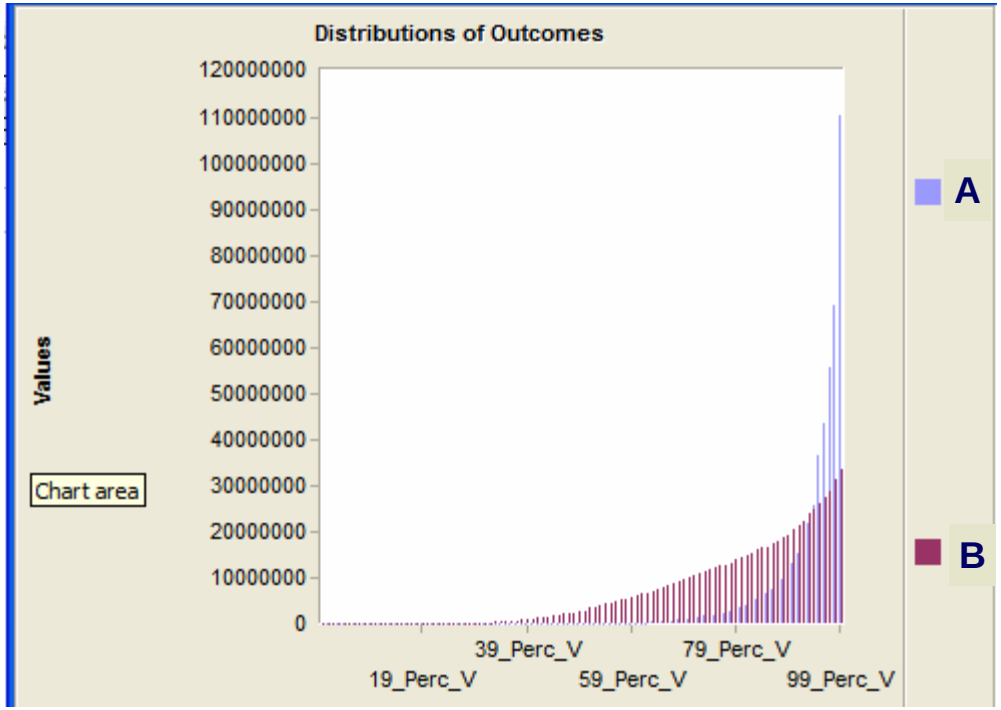
$$D(\downarrow) * P(\downarrow) = \frac{1}{1 + \text{risk free rate}} * P_{RN}(\downarrow)$$

Deflators put a greater emphasis on those outcomes in which risky assets perform badly

Deflators - Example

- PV of GMAB Payments
- Run A uses “Real World” scenarios, discounted using deflators
  - Mean = $6.5m
  - SDE = $0.8m
- Run B uses “Risk Neutral” scenarios, discounted at risk free rate
  - Mean = $7.2m
  - SDE = $0.2m
- Both runs give same value, within the tolerance of accuracy expected
  - Run A requires more scenarios for an accurate result

SDE = standard error, a measure of whether sufficient scenarios have been run



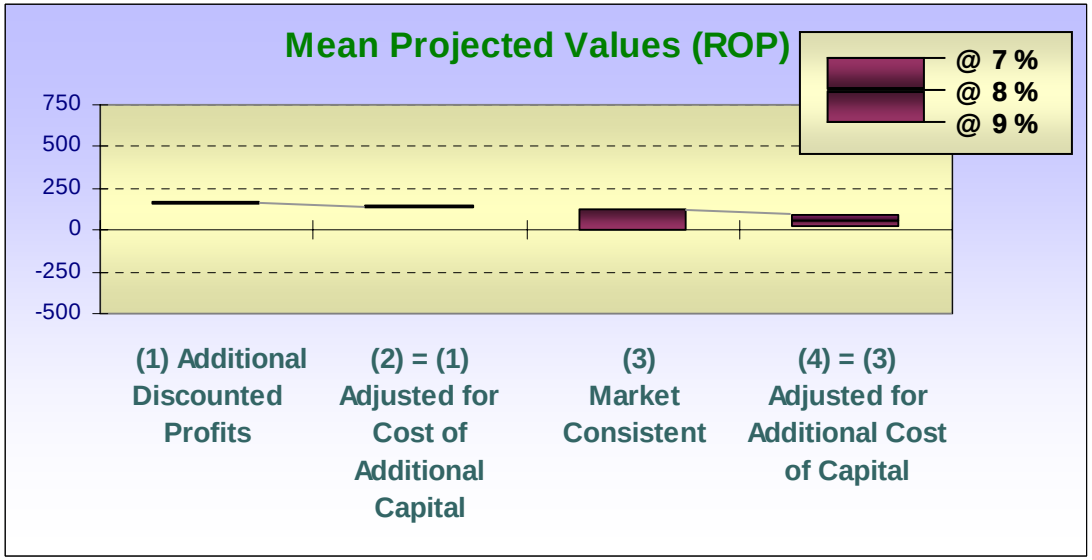
Note:

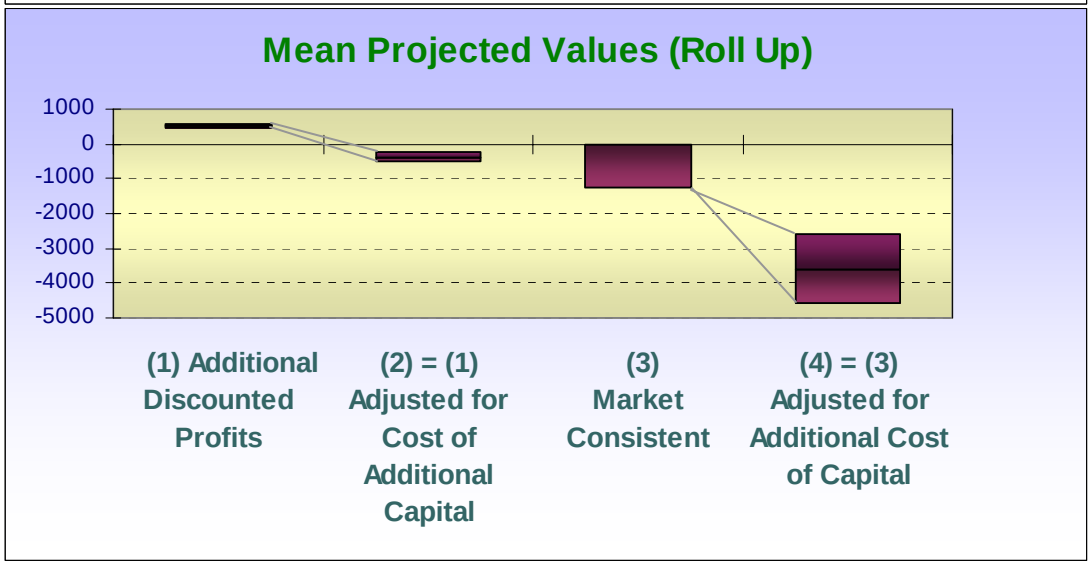
- If you know which scenarios pay out then, if using RW + deflators, you only need to run a relatively small proportion of the total scenarios to obtain an accurate result

Market Consistent Valuation - Summary

- Market consistent values can be calculated either:
  - Using “Real world” scenarios plus deflators, or
  - Using “Risk Neutral” scenarios, discounting at the risk free rate
- By “no arbitrage” reasoning, these values are also the cost of a perfect hedge, i.e.:
  - Assuming either that:
    - A replicating portfolio exists, or
    - A continuous dynamic hedging strategy is adopted
  - With:
    - No transaction costs
    - No lags

GMDB with ROP / Roll Up (revisited)





- Market consistent values for “ROP” appear slightly lower, but still profitable
- But market consistent values for “Roll Up” appear markedly worse
  - Reflecting their greater “riskiness”
  - Through greater weight being put on the scenarios that pay out
- Market consistent values need to be adjusted allow for cost of capital
  - Needs to be allowed for explicitly as a “loss of interest”

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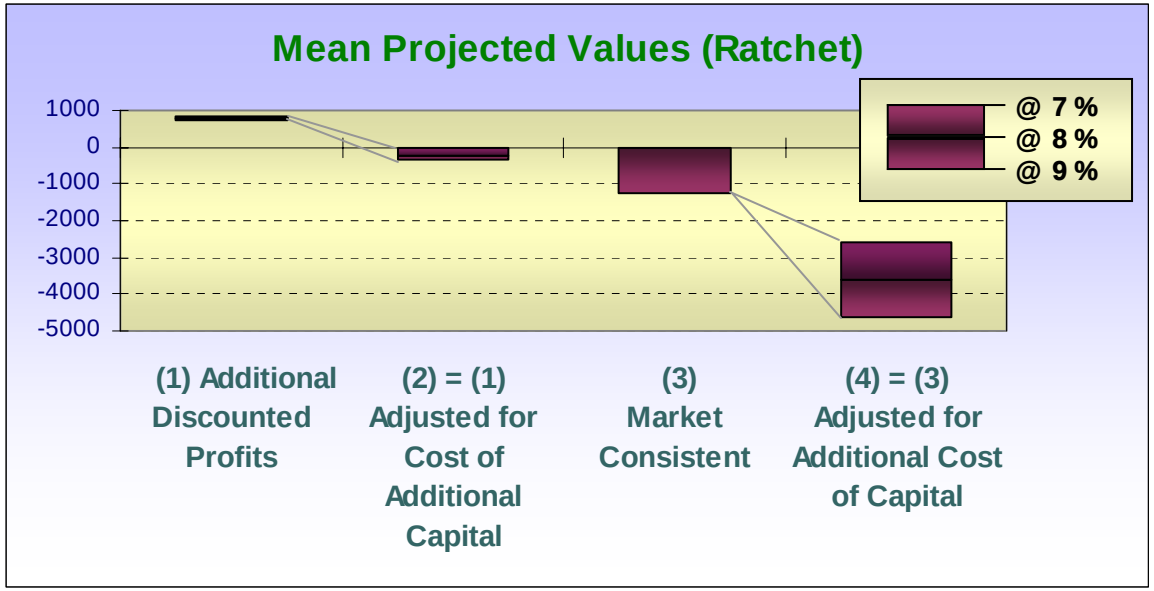
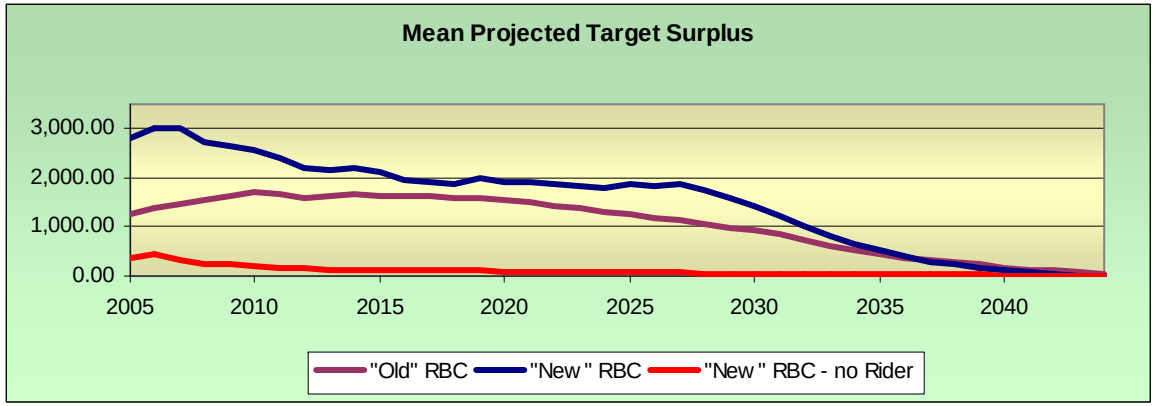
GMDB with Ratchet (30bps)





- Ratchet applies annually, on top of 5% roll up
- Value are very similar to Roll Up
  - But charge is 5bps more
- Ratchet causes target surplus to stay higher for longer

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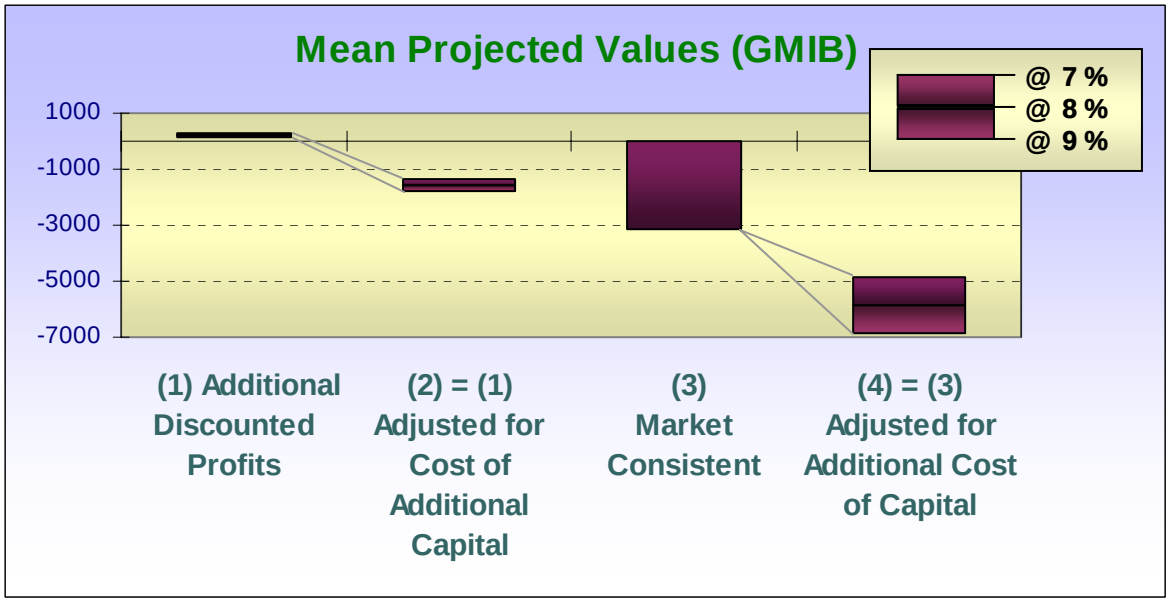
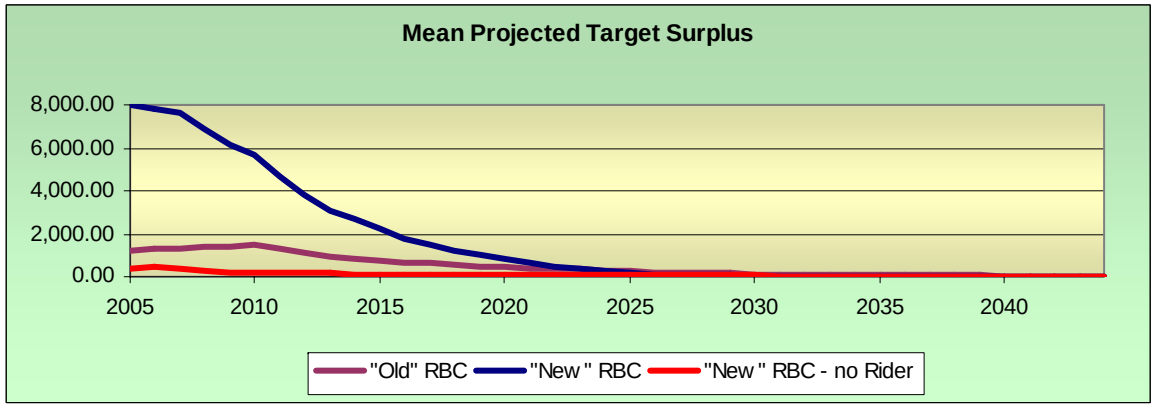
GMIB

- Annuitization is treated as a decrement
  - So that roll up of surplus for new RBC requirements does not include surplus from payout annuity
- Annuitization rate varies dynamically, based on “in the moneyness”
- Guarantee is:
  - Value of any excess of:
    - Premiums rolled up at 5% per year
    - Purchasing a lifetime annuity, guaranteed 5 years
    - Using 3% interest, ANN2000M
  - Over:
    - Account value, buying similar annuity at then current interest rates

GMIB (50bps)



- GMIB appears much more “risky”
  - Much greater capital requirement
  - Impact of loss of interest is very significant



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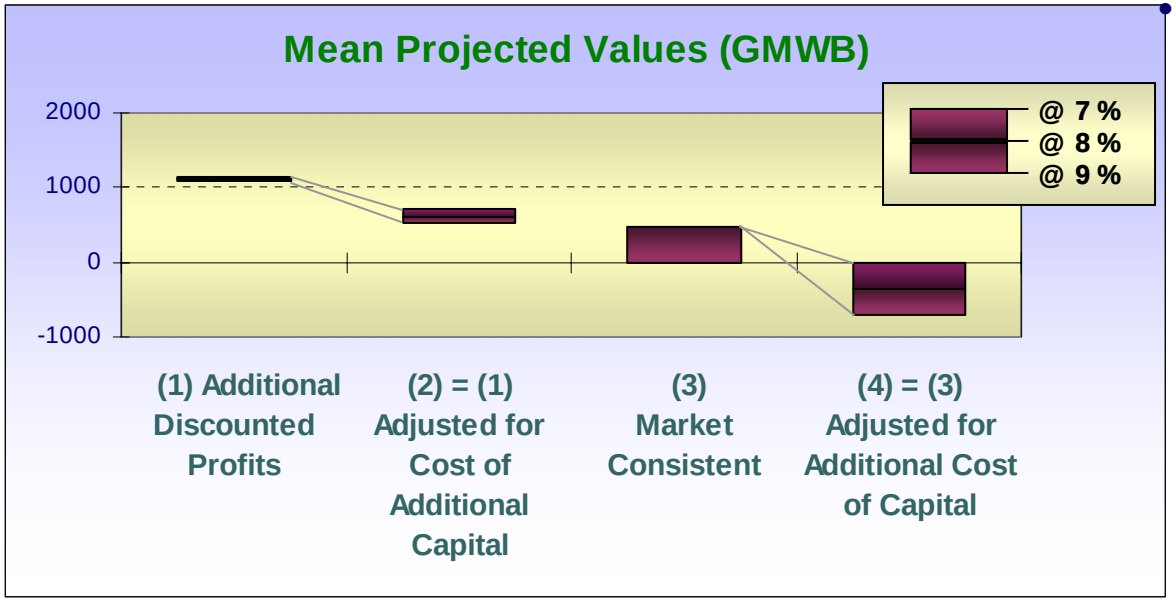
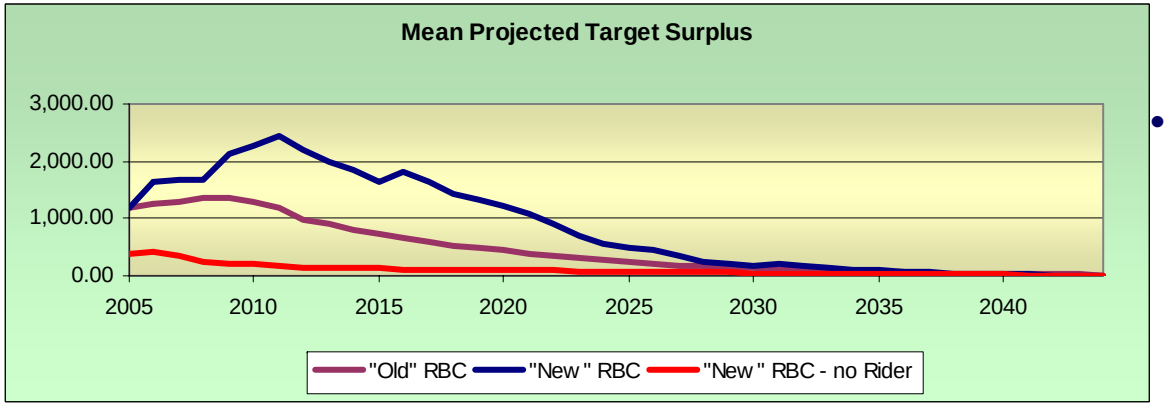
GMWB (50bps)





- Assumptions
  - Withdrawals, starting 4 years after issue
    - Varying between 1.75% and 7%, depending on “in the moneyness”
  - Guarantee is return of premium / reset amount
    - 5 yearly resets
    - Assumed to be taken when they would increase amount of withdrawal by at least 15%.
- Seems to be reasonably priced!
  - According to this model and set of assumptions

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Summary for Riders - Base Assumptions

	Charge (bps)	(1) Mean Additional Discounted Profits	(2) = (1) Adjusted for Cost of Additional Capital	(3) Market Consistent	(4) = (3) Adjusted for Cost of Additional Capital
GMDB (ROP)	10	162	139	130	58
GMDB (Roll up)	25	524	(388)	(1,270)	(3,588)
GMDB (Ratchet)	30	786	(206)	(1,241)	(3,606)
GMIB	50	189	(1,588)	(3,163)	(5,878)
GMWB	50	1,119	617	475	(357)

- Values assume cost of capital @ 8%
- On this data:
  - Some riders may need to be re-priced to reflect new “RBC” requirements
  - Question-mark as to whether the risk discount rate is sufficiently high to reflect the inherent riskiness of most riders

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Sensitivities (1) – Age at Issue

	Charge (bps)	Mean Additional Discounted Profits, Adjusted for Cost of Additional Capital		Market Consistent, Adjusted for Cost of Additional Capital	
		55	**65**	**55**	**65**
GMDB (ROP)	10	139	**(234)**	58	**(419)**
GMDB (Roll up)	25	(388)	**(1,779)**	(3,588)	**(5,584)**
GMDB (Ratchet)	30	(206)	**(1,868)**	(3,606)	**(5,729)**
GMIB	50	(1,588)	**(1,027)**	(5,878)	**(3,776)**
GMWB	50	617	**786**	(357)	**(33)**

- Higher age at issue:
  - Reduces values for GMDB, as would generally be expected
  - But increases values for GMIB and GMWB, again as would generally be expected

Sensitivities (2) – Investment

	Charge (bps)	Mean Additional Discounted Profits, Adjusted for Cost of Additional Capital		Market Consistent, Adjusted for Cost of Additional Capital	
		Diversified	**Concentrated**	**Diversified**	**Concentrated**
GMDB (ROP)	10	139	**70**	58	**(265)**
GMDB (Roll up)	25	(388)	**(1,058)**	(3,588)	**(5,075)**
GMDB (Ratchet)	30	(206)	**(937)**	(3,606)	**(5,140)**
GMIB	50	(1,588)	**(2,629)**	(5,878)	**(7,485)**
GMWB	50	617	**82**	(357)	**(1,592)**

- Greater concentration reduces value, as would be expected, as more risky
  - More apparent in the market consistent valuation

Sensitivities (3) – Dynamic Behavior

		Mean Additional Discounted Profits, Adjusted for Cost of Additional Capital		Market Consistent, Adjusted for Cost of Additional Capital	
		"On"	**"Off"**	**"On"**	**"Off"**
GMIB	Diversified	**(1,588)**	682	**(5,878)**	(581)
GMIB	Concentrated	**(2,629)**	433	**(7,485)**	(1,013)
GMWB	Diversified	**617**	653	**(357)**	455
GMWB	Concentrated	**82**	368	**(1,592)**	(183)

- "On" = with dynamic behavior, base case
- "Off" = without dynamic behavior

- Dynamic behavioral rules can make a great difference to the results
  - Particularly on a market consistent basis
  - Because they impact the downside scenarios more

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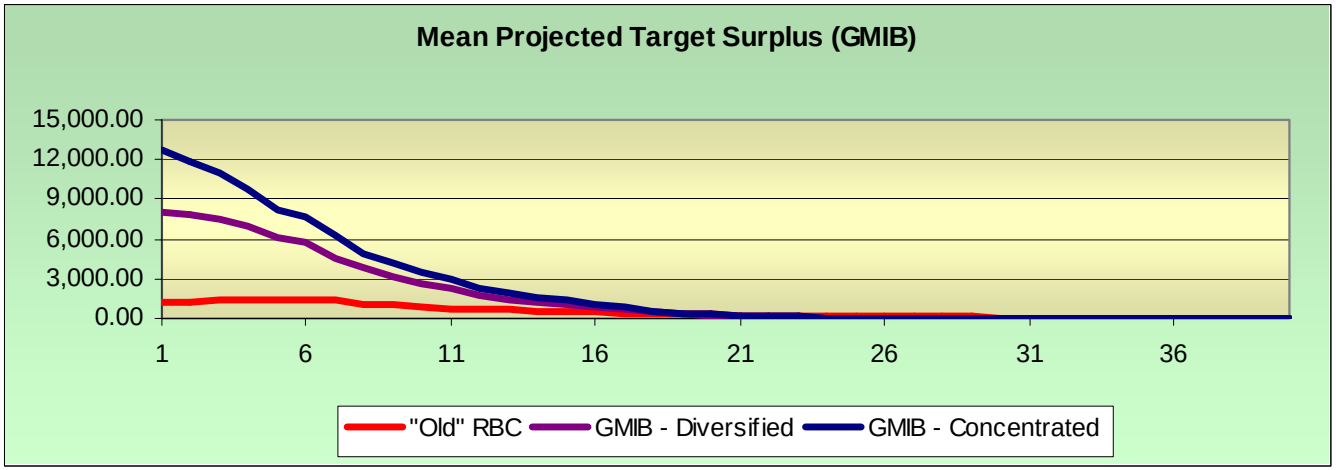
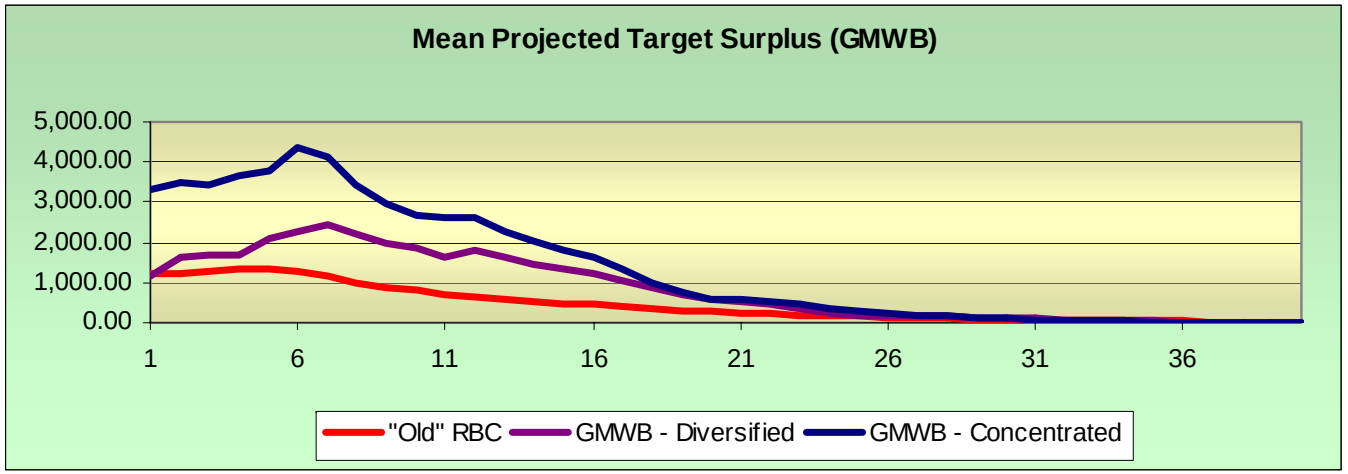
Another Look at Target Surplus





- Both GMIB and GMWB have greater capital requirements
  - Particularly GMIB

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VA Rider Case Study - Summary

- Methodology:
  - Discount at the required rate of return, or use market consistent valuation techniques?
  - Need to allow for cost of additional capital under RBC C3 Phase 2
  - Introduction to deflators
  - Introduction to stochastic on stochastic modeling
    - Here to stay
- Conclusions:
  - Pricing is sensitive, and in some cases very sensitive, both to the methodology and to the assumptions made!
  - Some riders may need to be re-priced
    - Both to reflect the new RBC requirements
    - And because use of a fixed discount (hurdle) rate may understate the risks

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